Impact of the Filter Cavity Length on the Cosmic Explorer Sensitivy

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1 Introduction

A filter cavity is used to compensate the rotation of the squeezed state caused by the ponderomotive squeezing of the interferometer [1–3]. In general, the maximum squeezing that can be observed for a given loss Λ and RMS phase noise θ_{rms} is, in terms of dB relative to unsqueezed vacuum [4],

$$N = 10 \log_{10}(\Lambda + 2\theta_{\mathrm{rms}}). \tag{1}$$

Therefore, as a rule of thumb, to reach the 10 dB squeezing target requires 8 % effective loss and 10 mrad of effective phase noise. For the nominal 4 km long CE filter cavity using the baseline parameters given in Table 1, both of these targets are exceeded—the filter cavity loss itself is about 10 % and the intrinsic dephasing alone is around 40 mrad—and so the filter cavity is responsible for limiting the squeezing improvement to only around 6 dB at low frequencies; see Fig. 1.

This note analyses how the length of the filter cavity affects the overall Cosmic Explorer sensitivity.[1] The main effect that the filter cavity length has on the sensitivity is indirectly through its connection to the filter cavity finesse: for the filter cavity to correctly compensate the ponderomotive squeezing of the interferometer, the filter cavity finesse must be inversely proportional to its length. Most noise sources attributable to the filter cavity are enhanced by its finesse[2] and so a longer filter cavity is generally favorable. A description of these noises and their quantitative impact on the sensitivity is given in Section 2, Section 3 analyzes the astrophysical performance of filter cavities of different lengths, and Section 4 summarizes some other effects and considerations that could be influenced by the filter cavity length.

The conclusion of this note is that it is unnecessary to make the CE filter cavity longer than the current baseline of 4 km and that, while possible to shorten it at the expense of limited astrophysical impact (c.f. Figs. 3 and 4 and Table 2), doing so will quickly make quantum noise the dominant noise source throughout most of the detection band (c.f. Fig. 5) and will decrease the limited margin if the parameters of Table 1 are not achieved or during times when the filter cavity cannot be optimally tuned for the interferometer state.

2 Squeezing degradations due to the filter cavity

The purpose of the filter cavity is to inject a frequency dependent squeezed state with the correct rotation to cancel the ponderomotive squeezing of the interferometer. The filter cavity is potentially responsible for three squeezing degradations: improperly rotating the squeezed state, pure loss, and phase noises [6, 7]. The filter cavity parameters needed to generate the correct rotation of the squeezed state—in particular its length, finesse, and detuning from resonance—determine the magnitude of the degradations from the remaining two mechanisms.

[1] The length of the A+ filter cavity was also studied in [5].

[2] Unlike the false propaganda that signal referred SEC loss is enhanced by the SEC finesse, filter cavity loss actually is enhanced by the filter cavity finesse.

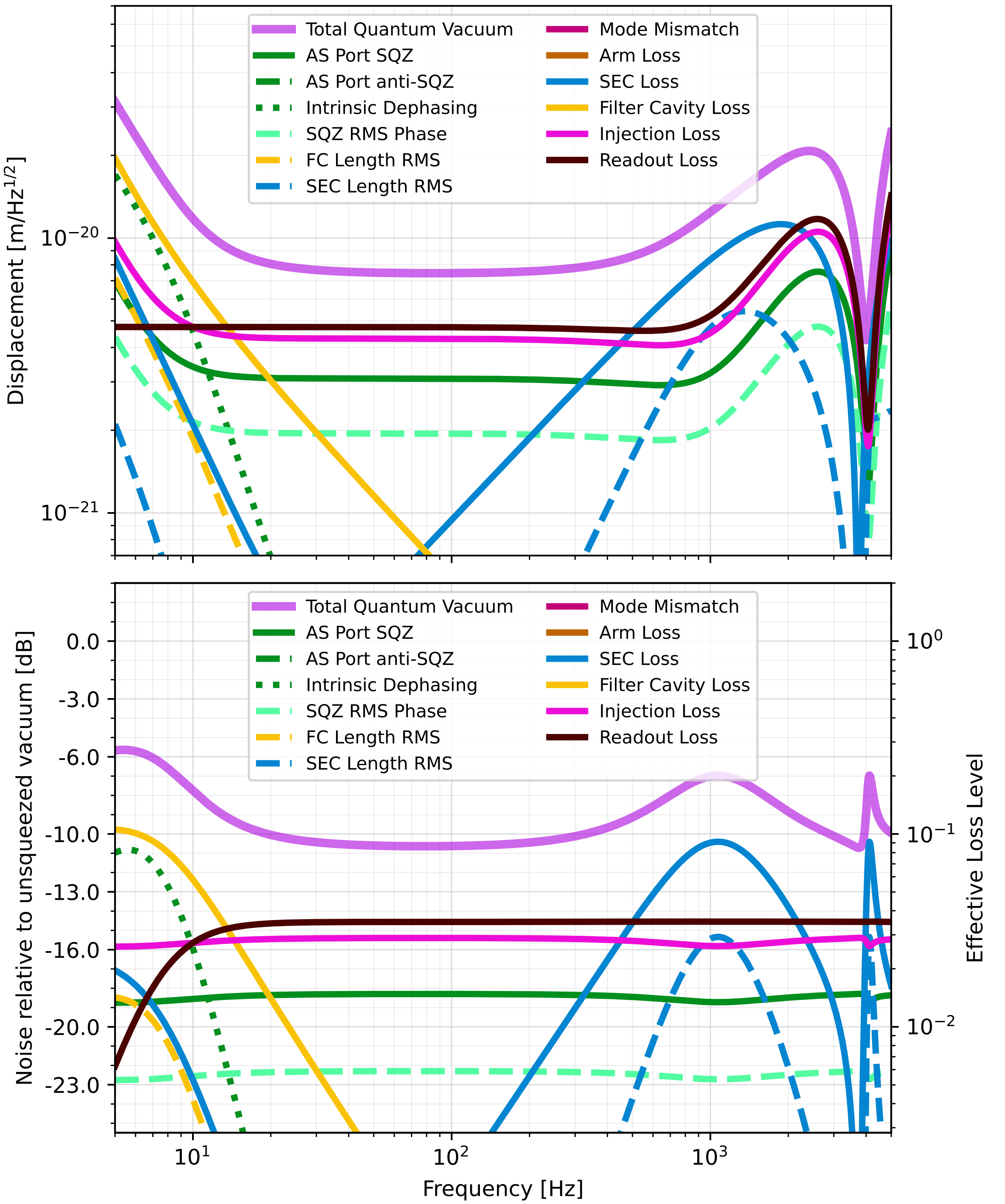

Figure 1: Quantum noise budgets using the parameters of Table 1. ***Top*:** Displacement noise; ***Bottom*:** noise relative to the unsqueezed vacuum that experiences the optomechanics of the interferometer, i.e. relative to the $\hbar\omega_0/2$ of vacuum entering the AS port and not to the $\hbar\omega_0/2$ of vacuum directly. The filter cavity is responsible for limiting the quantum improvement to only about 6 dB at low frequencies due to it's loss and phase noise (see Fig. 2).

Parameter	Symbol	Units	Value
FC length	L_{fc}	km	4
FC finesse	$\mathcal{F}_{\mathrm{fc}}$	—	3610
FC input coupler	T_{fc}	ppm	1700
FC detuning	$\Delta\omega_{\mathrm{fc}}/2\pi$	Hz	−5.2
FC round-trip loss	Λ_{fc}	ppm	80
FC RMS length	ΔL_{fc}	pm	1
Test mass mass	M	kg	320
ITM transmission	T_{i}	%	1.4
SEM transmission	T_{s}	%	2.0
Arm finesse	$\mathcal{F}_{\mathrm{a}}$	—	447
SEC finesse	$\mathcal{F}_{\mathrm{s}}$	—	313
Arm power	P_{a}	MW	1.5
Laser wavelength	λ	nm	1064
SQL frequency	$\Omega_{\mathrm{sql}}/2\pi$	Hz	7.3

Table 1: Baseline Cosmic Explorer parameters relevant to the filter cavity.

Note that the scaling relations derived in this section are valid for CE parameters other than the baseline ones given in Table 1 but should not be extrapolated to those of LIGO due to LIGO's low SEC finesse $\mathcal{F}_{\mathrm{s}} \approx 17.6$.

2.1 Squeezed state misrotation (misphasing)

The first degradation mechanism is if the filter cavity simply does not apply the correct rotation to cancel the interferometer's ponderomotive squeezing. In general, two filter cavities are needed to correctly do so [3]; however one filter cavity is sufficient if the optomechanical coupling of the interferometer, converting amplitude into phase fluctuations, is $\mathcal{K}_{\mathrm{ifo}} = (\Omega_{\mathrm{sql}}/\Omega)^2$ where

$$\Omega_{\mathrm{sql}} = \sqrt{\frac{16kP_{\mathrm{a}}}{Mc}\frac{\mathcal{F}_{\mathrm{a}}}{\mathcal{F}_{\mathrm{s}}}} \tag{2a}$$

$$\approx 2\pi \times 7.3\,\mathrm{Hz}\sqrt{\left(\frac{P_{\mathrm{a}}}{1.5\,\mathrm{MW}}\right)\left(\frac{320\,\mathrm{kg}}{M}\right)\left(\frac{\mathcal{F}_{\mathrm{a}}}{450}\right)\left(\frac{310}{\mathcal{F}_{\mathrm{s}}}\right)} \tag{2b}$$

is the SQL frequency [6]. The coupling in a dual-recycled Fabry-Perot Michelson interferometer, such as CE, is actually

$$\mathcal{K}_{\mathrm{ifo}} = \mathcal{K}_{\mathrm{mirror}}\, \mathrm{t}_{\mathrm{rse}}^2(\Omega) \approx \left(\frac{\Omega_{\mathrm{sql}}}{\Omega}\right)^2 \frac{1}{(1+\mathrm{i}\Omega/\gamma_{\mathrm{rse}})^2} \tag{3}$$

where $\mathcal{K}_{\mathrm{mirror}}$ is the optomechanical coupling of a mirror of mass $M/2$, $\mathrm{t}_{\mathrm{rse}}(\Omega)$ is the transfer function of a signal from the arm cavity to the AS port, and γ_{rse} is the DARM coupled cavity pole. The proper rotation can thus be applied to good approximation if $\gamma_{rse} \gg \Omega_{sql}$. This is the case for CE since $\gamma_{rse} \approx 825$ Hz, and so $\gamma_{rse}/\Omega_{sql} \approx 120$. This level is more than sufficient to keep anti-squeezing sub-dominant in CE, the misrotation is only about 0.1° around the SQL, however mode mismatch effects do have the potential to amplify this imperfect rotation [8]. Preliminary studies suggest that this will not be the case for CE and, in any case, the filter cavity length should not affect the magnitude of these effects.

In the absence of filter cavity losses, the optimal filter cavity bandwidth and detuning needed to compensate an optomechanical coupling $\mathcal{K}_{ifo} = (\Omega_{sql}/\Omega)^2$ are [6]

$$\gamma_{fc} = \frac{\pi c}{2L_{fc}\mathcal{F}_{fc}} = \Delta\omega_{fc} = \frac{\Omega_{sql}}{\sqrt{2}}. \tag{4}$$

The presence of realistic losses does not significantly change this result. Most of the degradation mechanisms associated with the filter cavity are enhanced by a factor of the filter cavity finesse $\mathcal{F}_{fc}$. It is therefore useful to have an expression for the finesse in terms of other parameters needed to keep the filter cavity bandwidth fixed at the optimal value:

$$\mathcal{F}_{fc} = \frac{\pi c}{2L_{fc}\gamma_{fc}} = \frac{\pi c}{\sqrt{2}L_{fc}\Omega_{sql}} \tag{5a}$$

$$\approx 3600\left(\frac{4\,\mathrm{km}}{L_{fc}}\right)\sqrt{\left(\frac{1.5\,\mathrm{MW}}{P_a}\right)\left(\frac{M}{320\,\mathrm{kg}}\right)\left(\frac{450}{\mathcal{F}_a}\right)\left(\frac{\mathcal{F}_s}{310}\right)} \tag{5b}$$

Equation (5) shows why longer filter cavities are generally beneficial. It also underscores the important point that the optimal finesse depends on the SQL frequency, which depends in turn on the arm power. While it is easy to change the detuning $\Delta\omega_{fc}$, changing the finesse requires changing the transmission of the filter cavity input coupler. Thus, if the interferometer arm power is not that for which the finesse was chosen according to Eq. (5), or if there are significant mode mismatch effects, more anti-squeezing will inevitably be observed over some frequency band. Indeed, for this reason, the A+ filter cavity could not be optimally tuned during the fourth observing run during which time this non-optimal behavior was observed [9].

Rather than swapping the mirror itself, one could imagine using a variable reflectivity [10] input coupler to tune the finesse in situ. Significant research would be needed in order to develop this option, but doing so would involve replacing the input coupler with another very short cavity. This would not affect the choice of the filter cavity length itself or grossly impact the vacuum system however.

2.2 Loss

The cavity enhanced filter cavity loss is

$$\Lambda = \frac{2\mathcal{F}_{fc}}{\pi}\frac{\Lambda_{fc}}{1+(\Omega/\gamma_{fc})^2} \approx \begin{cases} \dfrac{2\mathcal{F}_{fc}\Lambda_{fc}}{\pi} & \Omega \ll \gamma_{fc} \\ \dfrac{\mathcal{F}_{fc}\Lambda_{fc}}{\pi} & \Omega \approx \gamma_{fc} \\ \dfrac{\mathcal{F}_{fc}\Lambda_{fc}}{\pi}\left(\dfrac{\Omega_{sql}}{\Omega}\right)^2 & \Omega \gg \Omega_{sql} \end{cases} \tag{6a}$$

$$\approx 10\,\%\left(\frac{4\,\mathrm{km}}{L_{fc}}\right)\left(\frac{\Lambda_{fc}}{80\,\mathrm{ppm}}\right) \qquad \text{for } \Omega \approx \gamma_{fc} \tag{6b}$$

where Λ_{fc} is the round-trip filter cavity loss. This already exceeds the 8 % loss target needed to observe 10 dB of squeezing using the baseline parameters of Table 1. This expression also shows that it is the filter cavity loss per length Λ_{fc}/L_{fc}, rather than the loss Λ_{fc} itself, that directly matters to the cavity enhanced loss relevant to the overall detector sensitivity [6, 11]. Note that while Eq. (6) does not take radiation pressure into account (unlike the top panel of Fig. 1 which does), this is all that matters when comparing the squeezing enhancement (shown in the bottom panel of Fig. 1) since the optomechanical gain is common to all of the noises that source radiation pressure.

Loss caused by mode mismatch with the filter cavity is also a potential issue and is discussed in Section 4 where it is found unlikely to be a significant noise source.

2.3 Dephasing (phase noise)

A budget of several sources of phase noise, or dephasing, is shown in Fig. 2. The filter cavity is mainly responsible for the low frequency dephasing, which exceeds the target 10 mrad by about a factor of four, and is thus a large contributor towards the 6 dB limit to the low frequency squeezing.

When the upper and lower sidebands experience different loss, as they do in a detuned filter cavity, an intrinsic dephasing is generated [6, 7] which is given by [7]

$$\sqrt{\Xi} = \frac{2\mathcal{F}_{fc}}{\pi}\frac{\Lambda_{fc}\Omega\Delta\omega_{fc}/\gamma_{fc}^2}{\left[1+(\Omega-\Delta\omega_{fc})^2/\gamma_{fc}^2\right]\left[1+(\Omega+\Delta\omega_{fc})^2/\gamma_{fc}^2\right]} \tag{7}$$

$$\sqrt{\Xi}\Big|_{max} \approx 40\,\mathrm{mrad}\left(\frac{4\,\mathrm{km}}{L_{fc}}\right)\left(\frac{\Lambda_{fc}}{80\,\mathrm{ppm}}\right). \tag{8}$$

This dephasing can be understood by the fact that the noise in the upper and lower sidebands of a squeezed state is strictly increased, but in a correlated way so that the noise in one quadrature is reduced while the noise in the orthogonal quadrature is increased [12]. A phase noise is thus introduced when the sidebands experience different losses, which preserve the increased noise, while degrading the correlations responsible for defining the

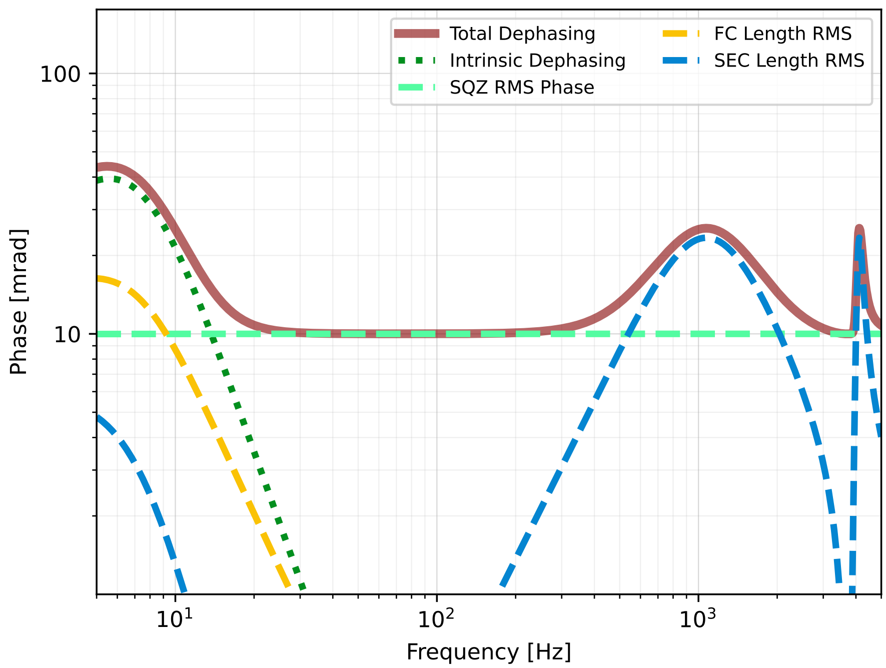

Figure 2: Dephasing budget using the parameters of Table 1. The large intrinsic dephasing and RMS length fluctuations due to the filter cavity are significant contributors to limiting the sensitivity improvement to only around 6 dB at low frequencies (see Fig. 1). Note that this budget is not a spectral density. The individual dephasing sources are correctly summed as described in Appendix B of [7].

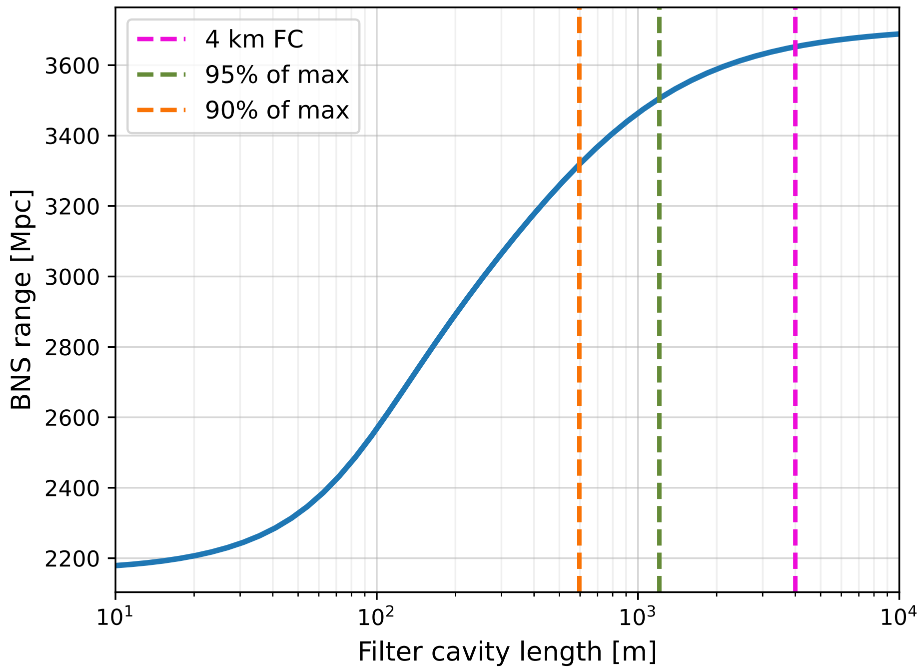

Figure 3: BNS range as a function of filter cavity length. Filter cavities of about 600 m and 1200 m in length have BNS ranges of 90 % and 95 %, respectively, of that of a filter cavity 10 km in length.

angle of the squeeze ellipse [7]. As with the loss Eq. (6), it is the loss per length $\Lambda_{\mathrm{fc}}/L_{\mathrm{fc}}$ that determines the magnitude of the intrinsic dephasing.

Since the rotation of the squeezed state is a function of the detunings of the cavities that it encounters as it propagates throughout the optical system, any fluctuations in those lengths will also be a source of phase noise [6, 7]. The dephasing due to the RMS length fluctuations of the filter cavity ΔL_{fc} is [13]

$$\sqrt{\Xi_{\Delta L}} = \frac{4\mathcal{F}_{\mathrm{fc}}}{\pi} \frac{k(\Omega/\gamma_{\mathrm{fc}})^2}{4 + (\Omega/\gamma_{\mathrm{fc}})^4} \Delta L_{\mathrm{fc}} \tag{9}$$

$$\sqrt{\Xi_{\Delta L}}\Big|_{\max} \approx 15\,\mathrm{mrad} \left(\frac{4\,\mathrm{km}}{L_{\mathrm{fc}}}\right) \left(\frac{\Delta L_{\mathrm{fc}}}{1\,\mathrm{pm}}\right). \tag{10}$$

The A+ filter cavity has achieved values of $\Delta L_{\mathrm{fc}} \lesssim 0.5\,\mathrm{pm}$ [1].

3 Astrophysical performance

Figure 3 shows the Cosmic Explorer BNS range as a function of filter cavity length. There is essentially no improvement in range as the filter cavity is extended beyond 10 km, and the 4 km long filter cavity almost reaches the maximum achievable range of about 3700 Mpc. A

Parameter	Units	Filter Cavity Length 600 m	1200 m	4 km	10 km
FC finesse	—	23 400	11 900	3610	1440
Min SQZ	dB	4.5	0.5	−5.5	−8.5
Max dephasing	mrad	270	140	45	20
BNS range	Mpc	3300	3500	3700	3700
BBH range	Mpc	6000	6100	6100	6100
Max detectable redshift	—	69	82	94	96

Table 2: Performance of different filter cavity lengths. BNS and BBH range estimates are for 1.4+1.4 $M_\odot$ and 30+30 $M_\odot$ optimally oriented non-spinning binary systems, respectively. "Min SQZ" is the minimum squeezing below vacuum (controlling for interferometer radiation pressure) rounded to the nearest half dB: increasingly negative values are increasingly better squeezing while positive values are anti-squeezing above vacuum; see Fig. 5. "Max dephasing" is the maximum total dephasing (intrinsic, RMS length, and RMS phase).

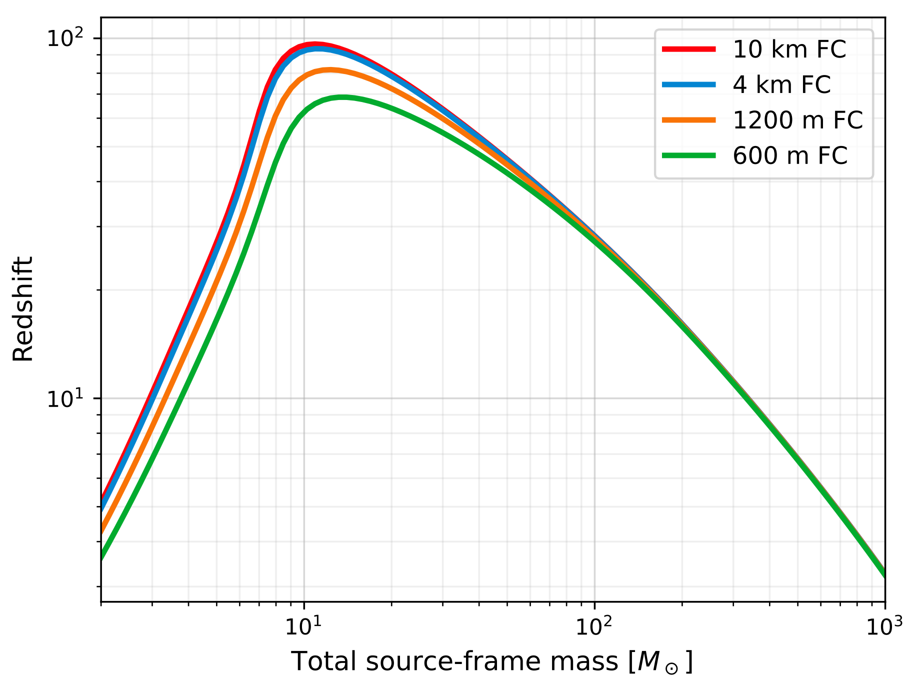

Figure 4: Horizon redshift at which the SNR in a detector is 8 for an optimally oriented, non-spinning, equal mass binary system.

1200 m filter cavity could reach 95 % of this maximum and a 600 m filter cavity could reach 90 % of the maximum.

Table 2 summarizes the performance of filter cavities of these four lengths; Fig. 4 shows the horizon redshift for these filter cavities. These results show that there is a very minimal difference in some measures of astrophysical performance between a 10 km and 4 km long filter cavity while there is a modest decrease as the filter cavity is shortened much below 4 km.

Table 2 also summarizes the performance of these filter cavities directly in terms of the sensitivity. Since quantum noise is not the dominant low frequency noise for the baseline design summarized in Table 1, the impact on the quantum noise itself as the filter cavity length is decreased is more severe. This is also shown in Fig. 5 where the sensitivities of these configurations are compared. Quantum noise has already become the dominant noise source throughout most of the detection band as the filter cavity length has been reduced to 1200 m, and the noise has actually been anti-squeezed *above* vacuum around the SQL at this length.

It is clear from these results that there are diminishing returns in increasing the filter cavity length beyond the baseline 4 km. While there is initially only a modest decrease in total sensitivity and astrophysical performance as the filter cavity length is further decreased, the overall impact will be more severe if the baseline 80 ppm round-trip loss or 1 pm RMS length fluctuation goals are not achieved.

4 Other effects and considerations

There are many other factors to consider when designing the filter cavity; see Ref. [5] for a detailed discussion of the A+ filter cavity. We here mention only a few that could be impacted by the filter cavity length.

Mode matching The noises considered in Section 2 are due to scalar losses, i.e. those where squeezed photons are lost and replaced with unsqueezed vacuum fluctuations. Mode mismatch effects, i.e. those due to squeezed photons being *coherently* scattered out of the fundamental mode into higher order modes (HOMs) and replaced with unsqueezed HOM vacuum, are also responsible for squeezing degradations [6, 7]. A mode mismatch of only 0.2 % was achieved between the A+ filter cavity and the squeezer [1]. Although coherent with the other sources of mismatch, in terms of pure loss this will always be well below the dominant quantum noises. A careful design of the CE filter cavity mode matching will be needed to achieve similar performance, but mode matching loss should not be a driver towards a longer filter cavity.

Similar to the effect described in [8] where squeezed state degradations occur around frequencies where HOMs become resonant in the arm cavities—which will certainly be a serious concern for CE—HOM resonances in the filter cavity can, in principle, also cause significant squeezing degradations[3] [14]. This has the potential to be a concern for the long

[3] Ref. [14] only discusses the anti-squeezing caused by HOM rotation and not the intrinsic dephasing due

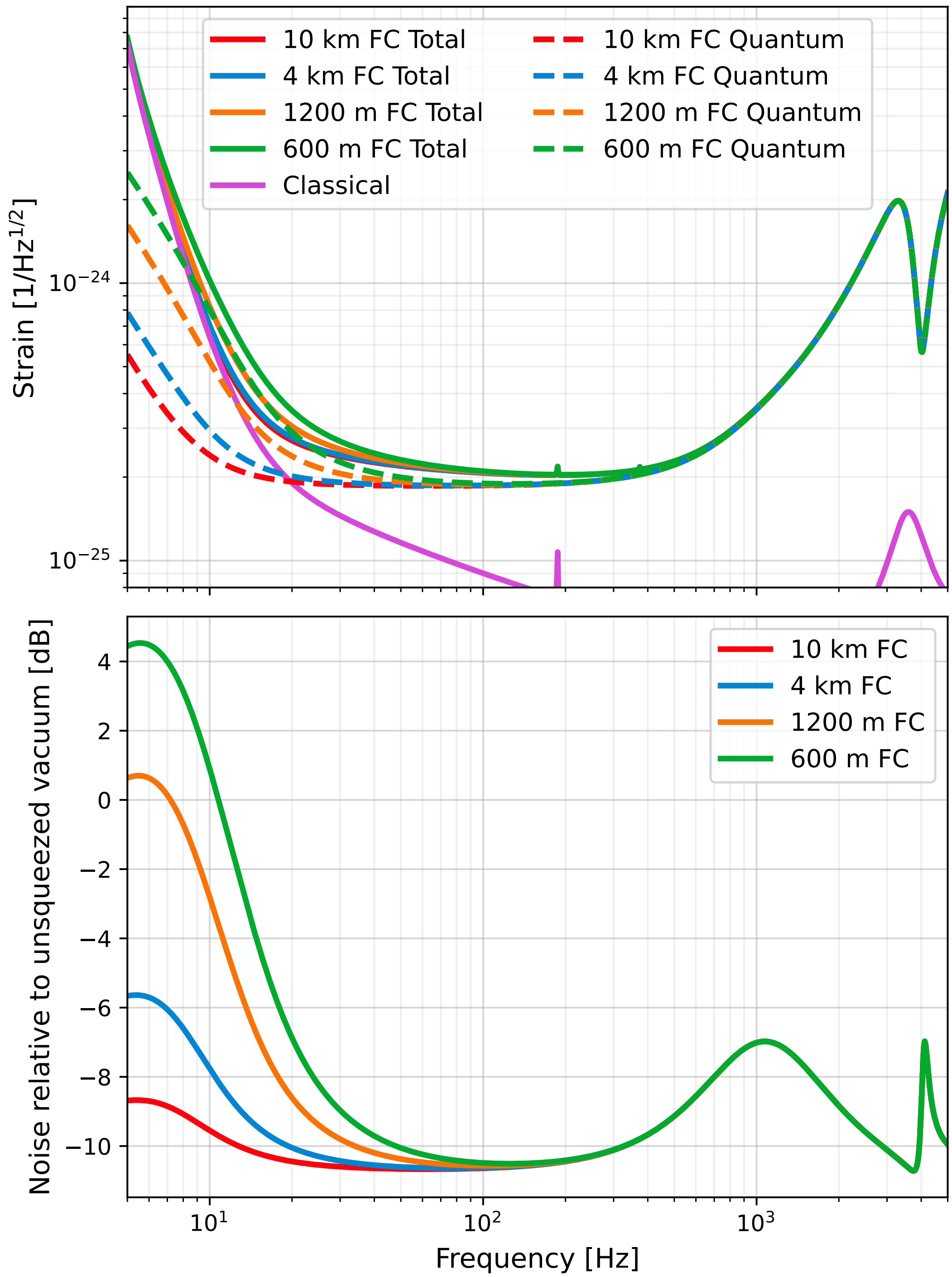

Figure 5: Comparison of the sensitivities of the filter cavities of Table 2. ***Top*:** Total strain noise (solid curves) and the quantum noise contributions (dashed curves) for each of these filter cavities. The classical noise is the same for all cases and is shown as solid purple. ***Bottom*:** Noise relative to unsqueezed vacuum for each case, after controlling for radiation pressure as in Fig. 1.

filter cavities considered by CE; however, the parameters discussed in [14] are unrealistically bad using an optical design engineered to cause problems. Using the phenomenological model of [7], we find it necessary to have of order 5 % mismatch with an exceptionally poor choice of Gouy phase in order for these effects to even begin to be noticeable with a 4 km long filter cavity. While more detailed studies are ongoing, the general phenomenological model of [7] shows excellent agreement with the more specific but detailed model of [8]—which as of yet cannot study these filter cavity effects—where they are both applicable and for small mismatch. We therefore find it unlikely that HOM resonances in the filter cavity would be a driver towards a shorter filter cavity.

Backscatter Due to the finite isolation of the Faraday isolators, some light exiting the dark port of the interferometer will travel backwards along the squeeze path, reflect off the filter cavity, and be subsequently backscattered into the interferometer along with the squeezed beam. This backscatter noise is also enhanced by the filter cavity finesse. Concerns about backscatter, given the LIGO levels of seismic isolation, resulted in LIGO A+ adding an extra Faraday isolator along the injection path [5], thereby increasing the injection losses. This decision was also made under estimates of the spurious backscattered light present when using DC readout; CE will instead use balanced homodyne detection which would result in about ten times less light than in DC readout according to the estimates of [5]. This needs to be checked quantitatively, but backscatter noise is unlikely to be a larger driver towards a longer filter cavity than the considerations discussed in Section 2 and, if it is a concern, will in any case set requirements on seismic isolation and spurious interferometer light instead.

Beamsize For the same cavity geometry, longer cavities have larger beams since the beamsize scales as $\sqrt{L_{\rm fc}}$. The largest beam in the 300 m A+ filter cavity is roughly 3 cm and so simply scaling that geometry up to the baseline 4 km CE filter cavity would result in a roughly 5.5 cm beam on the filter cavity end mirror. The largest beam in the 4 km long LIGO arm cavities is roughly 6 cm, and this is on the upper end of the largest beam which could fit in a cavity of this size. Baffles and the size of the filter cavity optics will need to be considered, but there should be no requirement on beamsize which limits the length of the filter cavity.

Control No attempt to analyze the control systems which will be needed to control the filter cavity have been made, but those details will be influenced by the cavity length and finesse [5]. However, these details should not affect the choice of length.

to mode mismatch, both are explicitly included in [7, 8], but their simulation method likely captured both effects in the numerical results presented.

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