Technical Note CE-T2400024-v1 2024/11/18

SEC Design Considerations and Processes

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This is an internal working note of the COSMIC EXPLORER project.

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1 Introduction

The SEC design process focuses on the design of the recycling cavity between the arm cavities and the output port of the interferometer. After having the appropriate system constraints, as well as choosing lengths and angles between mirrors, other mirror parameters can be derived.

2 Arm Cavities

The corner cavities must all start their design around the arm cavities; as such, the SEC design must start here as well. The CE arms are taken to be 40km, with a 12cm beamsize at the ITMs and ETMs. This permits two solutions for an eigenmode; however, for stabilization and control reasons the solution with a smaller beam waist is chosen.[1] From these considerations, the HR radius of curvature of the ITMs is derived:

$$\begin{aligned} q_{\text{ARM}} &= \frac{L_{\text{ARM}}}{2} + iz_R \\ \frac{1}{q_{\text{ARM}}} &= \frac{1}{R_c} - i\frac{\lambda}{\pi w_{\text{ARM}}^2} = \frac{z}{z^2+z_R^2} - i\frac{z_R}{z^2+z_R^2} \end{aligned} \tag{1}$$

$$\begin{aligned} z_R &= \frac{\pi w_{\text{ARM}}^2}{2\lambda} - \frac{1}{2}\sqrt{\frac{\pi^2 w_{\text{ARM}}^4}{\lambda^2} - L_{\text{ARM}}^2} \\ R_c &= \frac{L_{\text{ARM}}^2 + 4z_R^2}{2L_{\text{ARM}}} \end{aligned} \tag{2}$$

Parameter	Value	Unit
Arm Cavity Length	40	km
Arm Cavity Finesse	450	
Arm Cavity waist beamsize	68.986	mm
Arm Cavity q at ITM	$20+14.052i$	km
Beamsize at ITM	120	mm
Radius of Curvature of ITM HR	29.873	km

Table 1: Table of Values for CE arm cavities

The rest of the design process is centered around having a eigenmode in the SEC that follows design constraints while simultaneously matching this Arm Cavity eigenmode at the ITM.

3 SEC Design

Considering two corner layout configurations, the Long Crab 1 and Long Reverse aLIGO, we want to design the beam evolution from the ITMs to the SEM. Both layouts share the same number of optics as well as general layout, however their main distinction is the angle of incidence at the beampslitter.

[1] The smaller waist beamsize corresponds to negative g-factors of the cavity. When considering Sidles-Sigg instabilities, it is desirable to have the hard and soft modes be the major and minor modes, respectively, such that the frequency shift affect due to increasing power is not as severe for the soft mode[1].

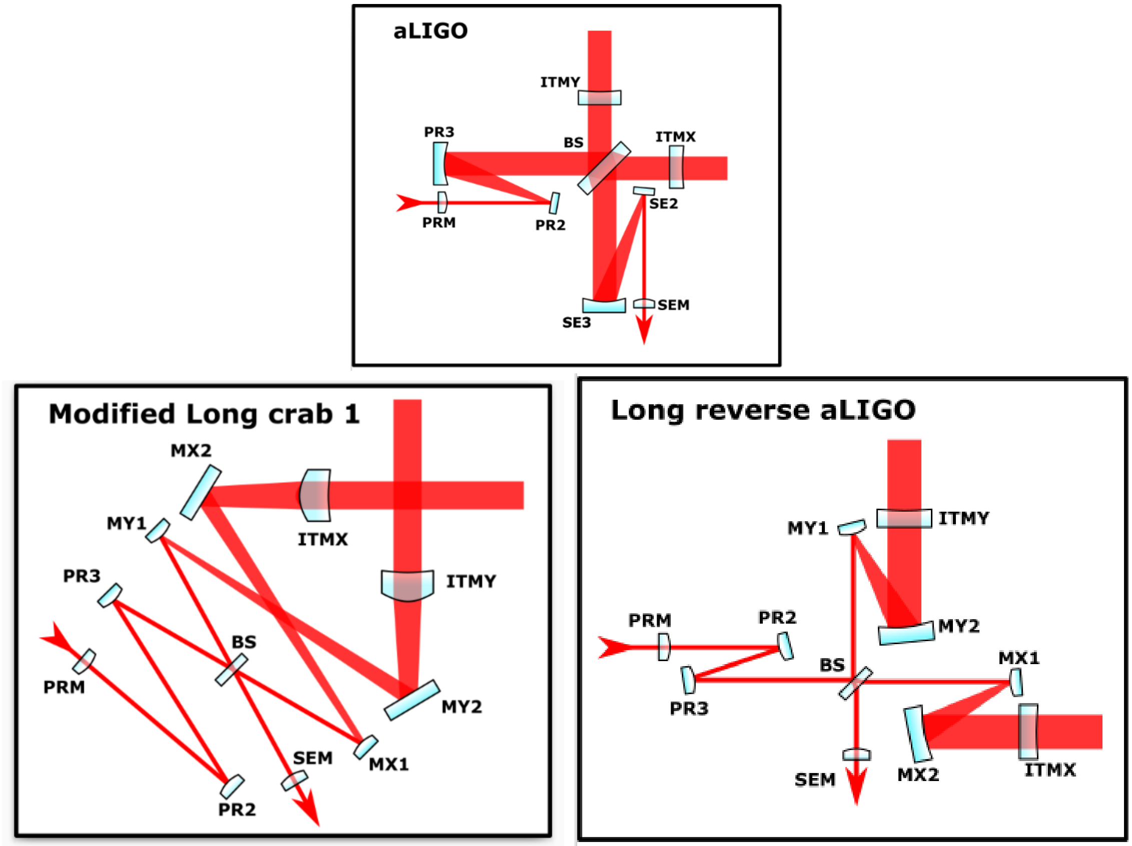

Figure 1: Cartoon sketches of proposed layouts for Cosmic Explorer

The design starts with the beam evolution through the beamsplitter to the first folding mirror. Then, the ITM lens (AR radius of curvature) and wedge angle, M2 radius of curvature, and M1 radius of curvature must be tuned to match the beam coming from the arm to the beam found through the beamsplitter.

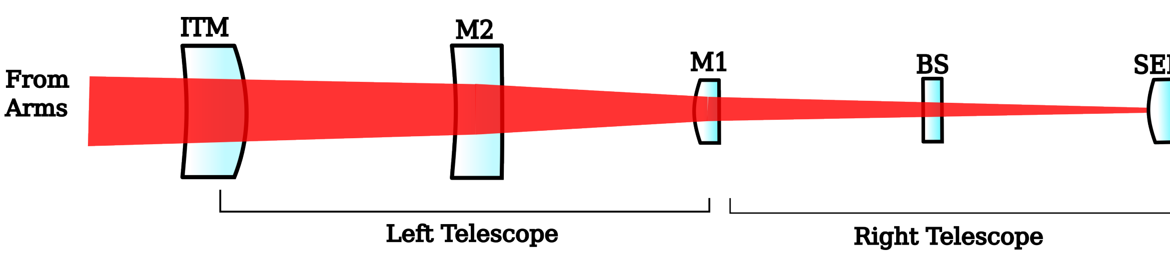

Figure 2: SEC as two telescopes

3.1 Right Telescoping Region: Signal Extraction Mirror and Beamsplitter

The telescoping region between the first folding mirror after the beamsplitter (M1) and the signal extraction mirror (SEM) can be completely characterized with a few parameters. The accumulated Gouy phase is one such parameter, as is motivated by mode sensing and control (20 deg). There is also a lower limit on the beamwidth at SEM (> 2 mm), motivated by lock loss power considerations. For thermal compensation reasons, a lower limit on the Rayleigh range through the beamsplitter is derived (> 1.5 m)[2]. By defining either the beamwidth at the beamsplitter or one of the distances separating the optics, we can derive the other characteristics of the telescope.

The q-parameter of the beam at the SEM can be derived, knowing the Rayleigh range and the beamwidth:

$$\begin{aligned} q_{\text{SEM}} &= z_{\text{SEM}} + i z_R \\ \frac{1}{q} &= \frac{z}{z^2 + z_R^2} - i\frac{z_R}{z^2 + z_R^2} \\ \Im\left(\frac{1}{q}\right) = -\frac{\lambda}{\pi w^2} &= -\frac{z_R}{z^2 + z_R^2} \\ z_{\text{SEM}} &= z_R\sqrt{\frac{\pi w_{\text{SEM}}^2}{\lambda z_R} - 1} \end{aligned} \tag{3}$$

Knowing the desired accumlated Gouy phase in the telescope, the total length can be derived.[2]

$$\begin{aligned} \tan(\psi_{\text{SEM}} - \psi_{\text{M1}}) \approx \tan\psi_{\text{SEC}} &= \frac{\tan(\psi_{\text{SEM}}) - \tan(\psi_{\text{M1}})}{1 + \tan(\psi_{\text{SEM}})\tan(\psi_{\text{M1}})} \\ &= z_R\frac{z_{\text{SEM}} - z_{\text{M1}}}{z_R^2 + z_{\text{SEM}} z_{\text{M1}}} \\ \frac{(z_R^2 + z_{\text{SEM}}^2)\tan(\psi_{\text{SEC}})}{z_R + z_{\text{SEM}}\tan(\psi_{\text{SEC}})} &= L_{\text{SEMtoBS}} + d_{\text{BS}} + L_{\text{BStoM1}} \end{aligned} \tag{4}$$

For thermo-refractive noise considerations it is also possible to derive a lower limit or desired beamwidth at the beamsplitter (>1 cm)[3], which further constrains our telescope, however this is a softer constraint can be done by defining the lengths instead:

$$L_{\text{SEMtoBS}} = z_R\sqrt{\frac{\pi w_{\text{BS}}^2}{\lambda z_R} - 1} + z_{\text{SEM}} \tag{5}$$

The last parameter that can be derived is the radius of curvature of the SEM, in order to make it a retro-reflector for the beam coming from the ITM, by matching the radius of curvature of the mirror to the curvature of the beam wavefront:

$$R_{\text{c,SEM}} = \frac{z_{\text{SEM}}^2 + z_R^2}{z_{\text{SEM}}} \tag{6}$$

As an example, with the general parameters defined we can characterize the telescope as shown in Table 2. Note that the values chosen are subject to change, however they represent

[2] Note, d_{BS} is the effective reduced thickness of the beamsplitter, dependent on the index of refraction and angle of incidence. It also needs to be doubled for the SEM to ITMX path, as it bounces off the HR side, traversing the substrate of the beamsplitter twice.

a parameter set close to the lower limits. This shows that if moderate values are chosen in focusing the beam in the left telescope, a reasonable minimum SEC length is around 100 meters. This is leads to a loss of about 8% of the optimal post-merger SNR in comparison to the optimal length of 20 meters for a nominal arm finesse of 450[4].

Parameter	Value	Unit
w_{SEM}	2	mm
ψ_{SEC}	20	deg
BS z_R	1.55	m
w_{BS}	18	mm
BS AoI	3	deg
q_{SEM}	-3.988 + 1.55i	m
$R_{\text{c,SEM}}$	-4.590	m
L_{SEMtoBS}	34.488	m
L_{BStoMY1}	33.084	m
L_{BStoMX1}	33.015	m

Table 2: Table of Values for SEM to M1 telescopes [derived values are in blue]. Note the BS thickness was assumed to be 10cm.

3.2 Left Telescoping Region: Input Test Mass to First Folding Mirror

From the above considerations, the q-parameter of the beam to the right of the first folding mirror (M1) is derived. The telescoping region to the left of the first folding mirror must then function to transform the q-parameter coming from the arms to M1. The parameters that can be tuned are the radii of curavture of M1, M2 and the AR side of the ITMs, as well as the angle of incidence on M1 and wedge angle of the ITM. From the angle of incidence at the beamsplitter and at M1, the angle of incidence at M2 is constrained to hold the arms perpendicular.

This is a more difficult minimization problem as the beam acquires astigmatism at each reflection off and transmission through each of the components in the left telescope, thus the mismatch to minimize is both between the left and right q-parameters at M1, along with the left sagittal and tangential q-parameters at M1.

First, this is accomplished by tuning the focusing parameters (ITM lens, M2 RoC) such that the beamwidth coming from the left matches that which was found at M1 in the right telescoping region. Then those values are used as an initial guess in a minimization function whose error function is the mismatch between the tangential and sagittal beams.

As of now, this has been done more effectievly for the Long Crab1 configuration, than the Long Reverse aLIGO; as the minimization functions seem to struggle with the Reverse aLIGO.

One derivation that is useful to note is the minimum wedge angle on the ITMs, whose value is used in the first minimization algorithm and can be slightly increased in the second:

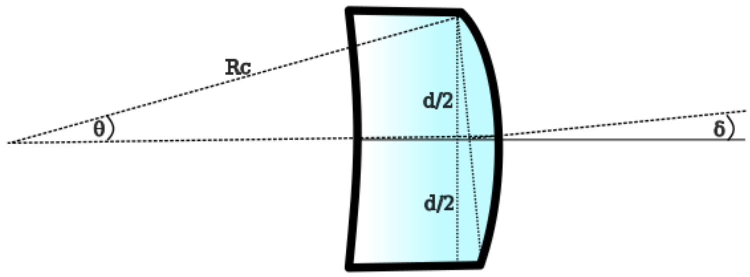

Figure 3: Wedge Angle relation to Radius of Curvature of AR side of ITM

$$\sin(\theta) = \frac{d/2}{R_c}$$
$$\delta > \theta = \arcsin\left(\frac{d/2}{R_c}\right) \tag{7}$$

This shows that the minimum wedge angle is determined such that the rays reflecting from the very edges of the curved surface are not reflected back to M2; this is done by ensuring the wedge angle δ is larger than "θ" which is the angle of the triangle formed by the radius of curvature R_c of the AR side and the radius of the mirror $d/2$.

3.3 Astigmatism Compensation

The width of parameter space that minimizes the beam astigmatism is much larger for the Crab layout, whose configuration has a characteristically larger angle of incidence on the second folding mirror M2 due to the reduced angle of incidence at the beamsplitter. In the reverse aLIGO layout this is reduced by the smaller angle of incidence on M2, and thus poses tighter constraints; this can be alleviated by allowing for toroidal mirrors, however these impose other design risks and challenges as well[5]. This also has an impact on the tolerances of related mirror parameters, namely the radius of

3.4 Implications and Designs

Figures 5 show "to-scale" renderings of the respective models using Fintrace[6], which combines the created finesse models with the associated gtrace models. The Long Crab1 model is very promising, as it has very good mode-matching to both X and Y arms as well as very low astigmatism in both paths. The current model for Long Reverse aLIGO, in contrast, does not have very good mode-matching while simulatenously having low astigmatism in both SECX and SECY. The associated values with each model can be found in Tables 3 and 4.

4 Summary and Tables of Values

From design constraints we can determine the beam evolution in each of the telescoping regions, and thus the required parameters of the mirrors in those regions. Once a model is

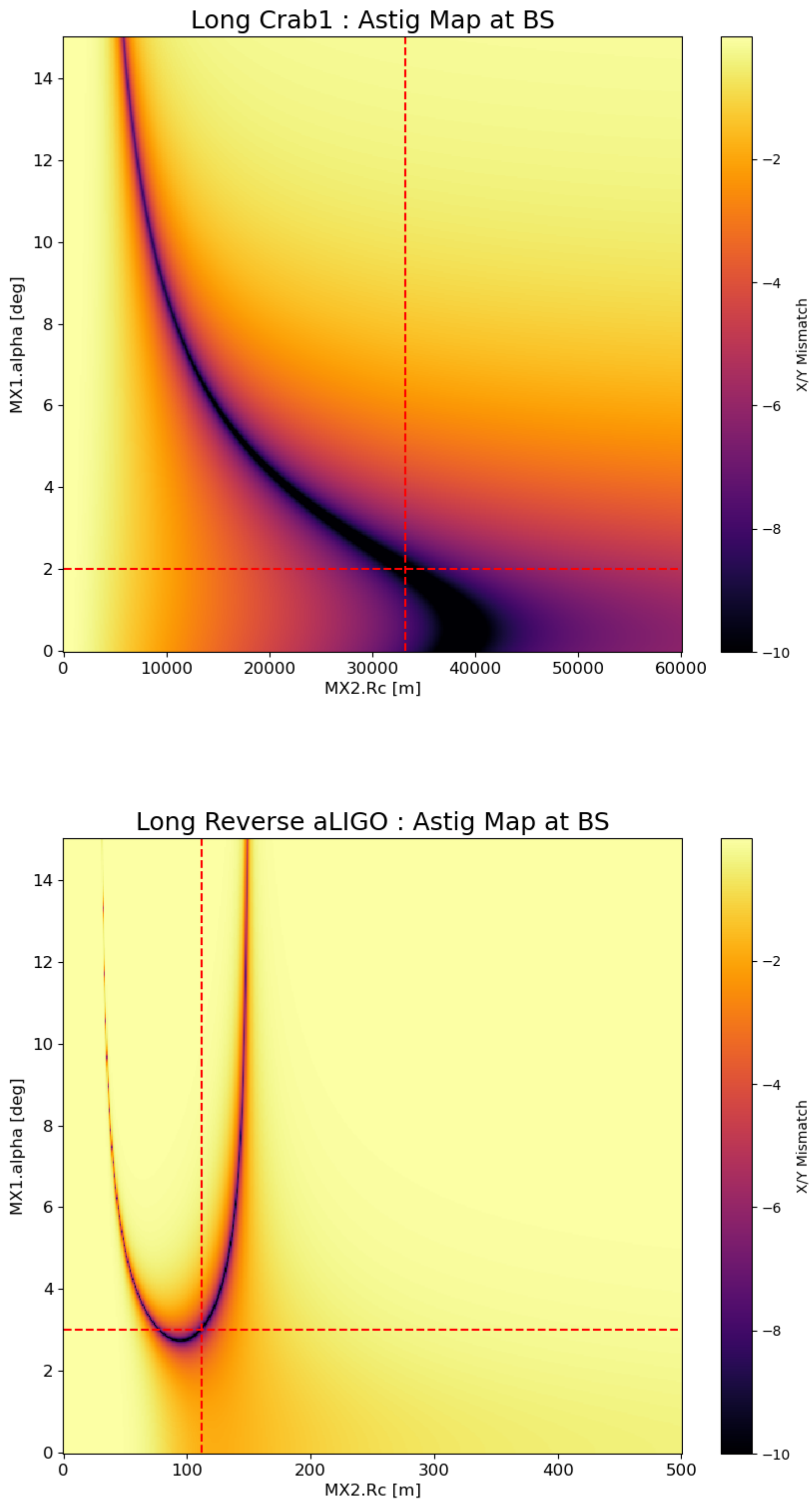

Figure 4: Astigmatism Map Across M1 AoI vs. M2 RoC for Long Crab1 [top] and Long Reverse aLIGO [bottom]

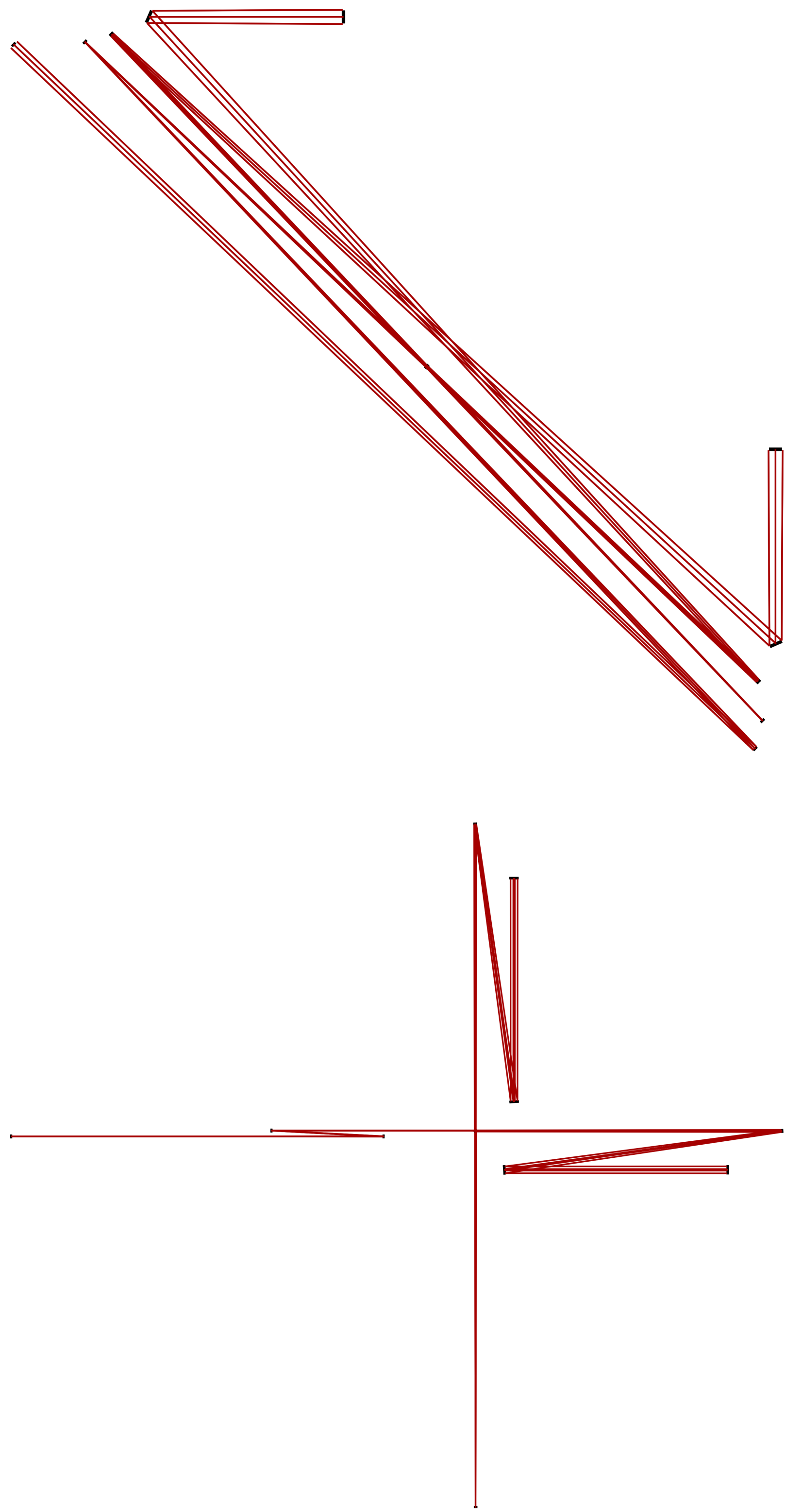

Figure 5: Geometric rendering of Long Crab1 and Long Reverse aLIGO, tuned for suitable vacuum layout.

Parameter	Value	Unit
SECY/ARMY Mismatch	0.207	ppm
SECY X/Y Mismatch	0.200	ppm
SECY X/Y Gouy Astig	-1.382	mdeg
SECX/ARMX Mismatch	0.131	ppm
SECX X/Y Mismatch	0.120	ppm
SECX X/Y Gouy Astig	1.030	mdeg
SECX/SECY Mismatch at BS	0.001	ppm

Table 3: Table of Values Analyzing Crab SEC design

Parameter	Value	Unit
SECY/ARMY Mismatch	37.586	ppm
SECY X/Y Mismatch	240.398	ppm
SECY X/Y Gouy Astig	-1.589	deg
SECX/ARMX Mismatch	5.119	ppm
SECX X/Y Mismatch	275.386	ppm
SECX X/Y Gouy Astig	-1.641	deg
SECX/SECY Mismatch at BS	471.122	ppm

Table 4: Table of Values Analyzing Reverse aLIGO SEC design

completed to this level, with mode-matching to the arms and astigmatism in both paths at reasonables levels, the design moves to understanding and tuning the interferometer for the appropriate frequency response.[7]

Parameter	x	y	Unit
ARMs			
ETM Front			
q	-20.000+14.052i	-20.000+14.052i	km
w	120.000	120.000	mm
R	-29.873	-29.873	km
w0	68.986	68.986	mm
kappa	6.03e-06	6.03e-06	ppm
ETM Front to ITM Front		40000.000	m
Accumulated Gouy Phase	54.908	54.908	deg
ITM Front			
q	20.000+14.052i	20.000+14.052i	km
w	120.000	120.000	mm
R	29.873	29.873	km
w0	68.986	68.986	mm
kappa	6.03e-06	6.03e-06	ppm
SECY			
ITMY HR Substrate			
q	24.201+11.716i	24.193+11.722i	km

w	120.055	120.001	mm
R	29.873	29.873	km
w0	52.312	52.326	mm
kappa	4.98e-06	4.98e-06	ppm
ITMY substrate		0.100	m
Accumulated Gouy Phase	0.000	0.000	deg
ITMY AR Substrate			
q	24.201+11.716i	24.193+11.722i	km
w	120.055	120.001	mm
R	29.873	29.873	km
w0	52.312	52.326	mm
kappa	4.98e-06	4.98e-06	ppm
ITMY Back			
q	-0.089+0.000i	-0.089+0.000i	km
w	120.055	119.991	mm
R	-89.102	-89.072	m
w0	0.251	0.251	mm
kappa	4.54e-01	4.54e-01	ppm
ITMY Back to MY2 Front		12.000	m
Accumulated Gouy Phase	0.009	0.009	deg
MY2 Front1			
q	-77.101+0.187i	-77.071+0.187i	m
w	103.886	103.825	mm
R	-77.102	-77.072	m
w0	0.251	0.251	mm
kappa	4.54e-01	4.54e-01	ppm
MY2 Front2			
q	-76.811+0.185i	-76.828+0.185i	m
w	103.886	103.825	mm
R	-76.812	-76.829	m
w0	0.250	0.251	mm
kappa	4.57e-01	4.57e-01	ppm
MY2 Front to MY1 Front		55.500	m
Accumulated Gouy Phase	0.186	0.187	deg
MY1 Front2			
q	-21.311+0.185i	-21.328+0.185i	m
w	28.824	28.824	mm
R	-21.313	-21.330	m
w0	0.250	0.251	mm
kappa	4.57e-01	4.57e-01	ppm
MY1 Front1			
q	-62.236+1.580i	-62.236+1.580i	m
w	28.824	28.824	mm
R	-62.276	-62.276	m
w0	0.732	0.732	mm

kappa	5.36e-02	5.36e-02	ppm
MY1 Front to BS Front1		28.147	m
Accumulated Gouy Phase	0.774	0.778	deg
BS Front1			
q	-34.089+1.580i	-34.089+1.580i	m
w	15.800	15.800	mm
R	-34.163	-34.163	m
w0	0.732	0.732	mm
kappa	5.36e-02	5.36e-02	ppm
BS Back			
q	-34.020+1.580i	-34.020+1.580i	m
w	15.768	15.768	mm
R	-34.094	-34.094	m
w0	0.732	0.732	mm
kappa	5.36e-02	5.36e-02	ppm
BS to SEM		30.000	m
Accumulated Gouy Phase	10.172	10.175	deg
SEM Front			
q	-4.021+1.580i	-4.020+1.580i	m
w	2.000	2.000	mm
R	-4.641	-4.641	m
w0	0.732	0.732	mm
kappa	5.36e-02	5.36e-02	ppm
SECX			
ITMX HR Substrate			
q	24.199+11.717i	24.193+11.722i	km
w	120.043	120.002	mm
R	29.873	29.873	km
w0	52.315	52.326	mm
kappa	4.98e-06	4.98e-06	ppm
ITMX substrate		0.100	m
Accumulated Gouy Phase	0.000	0.000	deg
ITMX AR Substrate			
q	24.199+11.717i	24.193+11.722i	km
w	120.044	120.002	mm
R	29.873	29.873	km
w0	52.315	52.326	mm
kappa	4.98e-06	4.98e-06	ppm
ITMX Back			
q	-0.089+0.000i	-0.089+0.000i	km
w	120.044	119.986	mm
R	-89.199	-89.152	m
w0	0.252	0.252	mm
kappa	4.53e-01	4.53e-01	ppm
ITMX Back to MX2 Front		12.000	m

Accumulated Gouy Phase	0.009	0.009	deg
MX2 Front1			
q	-77.198+0.187i	-77.152+0.187i	m
w	103.894	103.836	mm
R	-77.199	-77.153	m
w0	0.252	0.252	mm
kappa	4.53e-01	4.53e-01	ppm
MX2 Front2			
q	-76.809+0.185i	-76.826+0.185i	m
w	103.894	103.836	mm
R	-76.809	-76.826	m
w0	0.250	0.251	mm
kappa	4.57e-01	4.57e-01	ppm
MX2 Front to MX1 Front		55.500	m
Accumulated Gouy Phase	0.186	0.187	deg
MX1 Front2			
q	-21.309+0.185i	-21.326+0.185i	m
w	28.824	28.824	mm
R	-21.310	-21.327	m
w0	0.250	0.251	mm
kappa	4.57e-01	4.57e-01	ppm
MX1 Front1			
q	-62.236+1.580i	-62.236+1.580i	m
w	28.824	28.824	mm
R	-62.276	-62.276	m
w0	0.732	0.732	mm
kappa	5.36e-02	5.36e-02	ppm
MX1 Front to BS Back2		28.078	m
Accumulated Gouy Phase	0.772	0.775	deg
BS Back2			
q	-34.158+1.580i	-34.158+1.580i	m
w	15.832	15.832	mm
R	-34.231	-34.232	m
w0	0.732	0.732	mm
kappa	5.36e-02	5.36e-02	ppm
BS Back1			
q	-34.020+1.580i	-34.021+1.580i	m
w	15.768	15.768	mm
R	-34.094	-34.094	m
w0	0.732	0.732	mm
kappa	5.36e-02	5.36e-02	ppm
BS to SEM		30.000	m
Accumulated Gouy Phase	10.169	10.172	deg
SEM Front			
q	-4.020+1.580i	-4.021+1.580i	m

w	2.000	2.000	mm
R	-4.641	-4.641	m
w0	0.732	0.732	mm
kappa	5.36e-02	5.36e-02	ppm

Table 5: Table of Values Analyzing Crab Beam Parameters

Parameter	X Mirrors	Y Mirrors	Unit
	ETMX	**ETMY**	
T	5e-06	5e-06	
L	0.0	0.0	
Position	[39994.942, 22.704]	[21.544, 39994.922]	m
theta	180.002	270.000	deg
Diameter	70.0	70.0	cm
Thickness	200.0	200.0	mm
RcHR	2.9873e+04	2.9873e+04	m
RcAR	inf	inf	m
AoI	0.00	0.00	deg
Wedge	0.00	0.00	deg
	ITMX	**ITMY**	
T	0.014	0.014	
L	0.0	0.0	
Position	[-5.058, 21.521]	[21.544, -5.078]	m
theta	0.002	90.000	deg
Diameter	70.0	70.0	cm
Thickness	100.0	100.0	mm
RcHR	2.9873e+04	2.9873e+04	m
RcAR	3.9974e+01	3.9928e+01	m
AoI	0.00	0.00	deg
Wedge	0.90	0.72	deg
	MX2	**MY2**	
T	5e-06	5e-06	
L	0.0	0.0	
Position	[-17.058, 21.521]	[21.544, -17.078]	m
theta	-23.748	113.750	deg
Diameter	70.0	70.0	cm
Thickness	100.0	100.0	mm
RcHR	3.3238e+04	4.4607e+04	m
RcAR	inf	inf	m
AoI	23.75	23.75	deg
Wedge	0.00	0.00	deg
	MX1	**MY1**	
T	5e-06	5e-06	
L	0.0	0.0	

Position	[20.439, -19.397]	[-19.375, 20.417]	m
theta	134.501	315.500	deg
Diameter	20.0	20.0	cm
Thickness	100.0	100.0	mm
RcHR	-6.4831e+01	-6.4842e+01	m
RcAR	inf	inf	m
AoI	2.00	2.00	deg
Wedge	0.00	0.00	deg
	BS		
T	0.5		
L	0.0		
Position	[0.000, 0.000]		m
theta	135.000		deg
Diameter	20.0		cm
Thickness	100.0		mm
RcHR	inf		m
RcAR	inf		m
AoI	1.50		deg
Wedge	0.00		deg
	SEM		
T	0.02		
L	0.0		
Position	[20.720, -21.833]		m
theta	133.500		deg
Diameter	20.0		cm
Thickness	50.0		mm
RcHR	-4.6414e+00		m
RcAR	inf		m
AoI	0.00		deg
Wedge	0.00		deg

Table 6: Table of Values Analyzing Crab Mirror Parameters

Parameter	x	y	Unit
	ARMs		
ETM Front			
q	-20.000+14.052i	-20.000+14.052i	km
w	120.000	120.000	mm
R	-29.873	-29.873	km
w0	68.986	68.986	mm
kappa	6.03e-06	6.03e-06	ppm
ETM Front to ITM Front		40000.000	m
Accumulated Gouy Phase	54.908	54.908	deg
ITM Front			

q	20.000+14.052i	20.000+14.052i	km
w	120.000	120.000	mm
R	29.873	29.873	km
w0	68.986	68.986	mm
kappa	6.03e-06	6.03e-06	ppm
SECY			
ITMY HR Substrate			
q	24.079+11.811i	23.784+12.034i	km
w	119.267	117.431	mm
R	29.873	29.873	km
w0	52.525	53.017	mm
kappa	4.94e-06	4.85e-06	ppm
ITMY substrate		0.100	m
Accumulated Gouy Phase	0.001	0.001	deg
ITMY AR Substrate			
q	24.079+11.811i	23.784+12.034i	km
w	119.267	117.432	mm
R	29.873	29.873	km
w0	52.525	53.017	mm
kappa	4.94e-06	4.85e-06	ppm
ITMY Back			
q	-0.123+0.000i	-0.122+0.000i	km
w	119.267	117.355	mm
R	-122.644	-122.339	m
w0	0.348	0.353	mm
kappa	2.36e-01	2.30e-01	ppm
ITMY Back to MY2 Front		20.000	m
Accumulated Gouy Phase	0.403	0.402	deg
MY2 Front1			
q	-102.643+0.358i	-102.338+0.368i	m
w	99.818	98.170	mm
R	-102.644	-102.339	m
w0	0.348	0.353	mm
kappa	2.36e-01	2.30e-01	ppm
MY2 Front2			
q	-36.523+0.045i	-36.549+0.047i	m
w	99.818	98.170	mm
R	-36.523	-36.549	m
w0	0.124	0.126	mm
kappa	1.87e+00	1.80e+00	ppm
MY2 Front to MY1 Front		25.000	m
Accumulated Gouy Phase	2.299	2.291	deg
MY1 Front2			
q	-11.523+0.045i	-11.549+0.047i	m
w	31.493	31.021	mm

R	-11.524	-11.549	m
w0	0.124	0.126	mm
kappa	1.87e+00	1.80e+00	ppm
MY1 Front1			
q	-68.093+1.584i	-68.073+1.632i	m
w	31.493	31.021	mm
R	-68.129	-68.112	m
w0	0.732	0.743	mm
kappa	5.34e-02	5.19e-02	ppm
MY1 Front to BS Front1		30.515	m
Accumulated Gouy Phase	14.307	14.249	deg
BS Front1			
q	-37.577+1.584i	-37.558+1.632i	m
w	17.391	17.126	mm
R	-37.644	-37.629	m
w0	0.732	0.743	mm
kappa	5.34e-02	5.19e-02	ppm
BS Back			
q	-37.526+1.584i	-37.479+1.632i	m
w	17.367	17.090	mm
R	-37.592	-37.550	m
w0	0.732	0.743	mm
kappa	5.34e-02	5.19e-02	ppm
BS to SEM		33.549	m
Accumulated Gouy Phase	23.554	23.884	deg
SEM Front			
q	-3.976+1.584i	-3.930+1.632i	m
w	1.979	1.939	mm
R	-4.607	-4.607	m
w0	0.732	0.743	mm
kappa	5.34e-02	5.19e-02	ppm
SECX			
ITMX HR Substrate			
q	24.234+11.690i	23.924+11.930i	km
w	120.272	118.292	mm
R	29.873	29.873	km
w0	52.253	52.787	mm
kappa	5.00e-06	4.89e-06	ppm
ITMX substrate		0.100	m
Accumulated Gouy Phase	0.001	0.001	deg
ITMX AR Substrate			
q	24.234+11.690i	23.924+11.930i	km
w	120.272	118.293	mm
R	29.873	29.873	km
w0	52.253	52.787	mm

kappa	5.00e-06	4.89e-06	ppm
ITMX Back			
q	-0.126+0.000i	-0.125+0.000i	km
w	120.272	118.213	mm
R	-125.647	-125.325	m
w0	0.354	0.359	mm
kappa	2.29e-01	2.22e-01	ppm
ITMX Back to MX2 Front		20.000	m
Accumulated Gouy Phase	0.409	0.407	deg
MX2 Front1			
q	-105.646+0.370i	-105.323+0.381i	m
w	101.128	99.348	mm
R	-105.647	-105.325	m
w0	0.354	0.359	mm
kappa	2.29e-01	2.22e-01	ppm
MX2 Front2			
q	-36.595+0.044i	-36.621+0.046i	m
w	101.128	99.348	mm
R	-36.595	-36.621	m
w0	0.123	0.125	mm
kappa	1.91e+00	1.84e+00	ppm
MX2 Front to MX1 Front		25.000	m
Accumulated Gouy Phase	2.316	2.306	deg
MX1 Front2			
q	-11.595+0.044i	-11.621+0.046i	m
w	32.042	31.527	mm
R	-11.595	-11.622	m
w0	0.123	0.125	mm
kappa	1.91e+00	1.84e+00	ppm
MX1 Front1			
q	-68.110+1.531i	-68.118+1.582i	m
w	32.042	31.527	mm
R	-68.144	-68.154	m
w0	0.720	0.732	mm
kappa	5.53e-02	5.35e-02	ppm
MX1 Front to BS Back2		30.432	m
Accumulated Gouy Phase	14.283	14.213	deg
BS Back2			
q	-37.678+1.531i	-37.686+1.582i	m
w	17.735	17.453	mm
R	-37.740	-37.752	m
w0	0.720	0.732	mm
kappa	5.53e-02	5.35e-02	ppm
BS Back1			
q	-37.574+1.531i	-37.528+1.582i	m

w	17.687	17.380	mm
R	-37.637	-37.594	m
w0	0.720	0.732	mm
kappa	5.53e-02	5.35e-02	ppm
BS to SEM		33.549	m
Accumulated Gouy Phase	23.530	23.848	deg
SEM Front			
q	-4.025+1.531i	-3.978+1.582i	m
w	2.025	1.981	mm
R	-4.607	-4.607	m
w0	0.720	0.732	mm
kappa	5.53e-02	5.35e-02	ppm

Table 7: Table of Values Analyzing Reverse aLIGO Beam Parameters

Parameter	X Mirrors	Y Mirrors	Unit
	ETMX	**ETMY**	
T	5e-06	5e-06	
L	0.0	0.0	
Position	[40025.683, -2.644]	[2.613, 40025.652]	m
theta	180.000	270.000	deg
Diameter	70.0	70.0	cm
Thickness	200.0	200.0	mm
RcHR	2.9873e+04	2.9873e+04	m
RcAR	inf	inf	m
AoI	0.00	0.00	deg
Wedge	0.00	0.00	deg
	ITMX	**ITMY**	
T	0.014	0.014	
L	0.0	0.0	
Position	[25.683, -2.644]	[2.613, 25.652]	m
theta	0.000	90.000	deg
Diameter	70.0	70.0	cm
Thickness	100.0	100.0	mm
RcHR	2.9873e+04	2.9873e+04	m
RcAR	5.6248e+01	5.4910e+01	m
AoI	0.00	0.00	deg
Wedge	2.00	1.97	deg
	MX2	**MY2**	
T	5e-06	5e-06	
L	0.0	0.0	
Position	[5.683, -2.644]	[2.613, 5.652]	m
theta	3.000	93.000	deg
Diameter	70.0	70.0	cm

Thickness	100.0	100.0	mm
RcHR	1.1213e+02	1.1355e+02	m
RcAR	inf	inf	m
AoI	3.00	3.00	deg
Wedge	0.00	0.00	deg
	MX1	**MY1**	
T	5e-06	5e-06	
L	0.0	0.0	
Position	[30.546, -0.031]	[0.000, 30.515]	m
theta	183.000	273.000	deg
Diameter	20.0	20.0	cm
Thickness	100.0	100.0	mm
RcHR	-2.7983e+01	-2.7777e+01	m
RcAR	inf	inf	m
AoI	3.00	3.00	deg
Wedge	0.00	0.00	deg
	BS		
T	0.5		
L	0.0		
Position	[0.000, 0.000]		m
theta	135.000		deg
Diameter	20.0		cm
Thickness	100.0		mm
RcHR	inf		m
RcAR	inf		m
AoI	45.00		deg
Wedge	0.00		deg
	SEM		
T	0.02		
L	0.0		
Position	[0.031, -33.664]		m
theta	90.000		deg
Diameter	20.0		cm
Thickness	50.0		mm
RcHR	-4.6074e+00		m
RcAR	inf		m
AoI	0.00		deg
Wedge	0.00		deg

Table 8: Table of Values Analyzing Reverse aLIGO Mirror Parameters

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