THE COSMIC EXPLORER GRAVITATIONAL WAVE OBSERVATORY

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Thermorefractive Noise Limitation on Cosmic Explorer's Beamsplitter Size

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1 Introduction

This note summarizes the beam size and optic thickness limitations on the Cosmic Explorer beamsplitter arising from thermorefractive noise. It is most limiting at 15 Hz. Equation 8 provides the thermorefractive noise in the readout as a function of arm finesse, beamsplitter thickness and beam size on the beamsplitter.

2 Formula

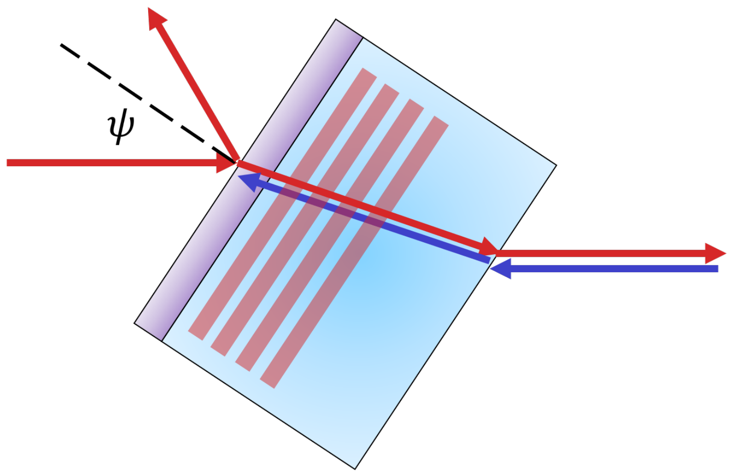

Figure 1: Breif schematic of a beamsplitter. Red arrows are forward beams, and blue arrows are backward beams. ψ denotes the angle of incidence.

The beamsplitter (BS) in a laser interferometer induces the thermorefractive (TR) noise. The light incident on the BS has a finite angle of incidence, and a standing wave is formed inside the BS. In order to estimate the TR noise level, they need to be taken into account.

The TR noise introduced by the BS can be expressed as [1]

$$S^{\mathrm{TR}}(\omega) = \frac{4k_{\mathrm{B}}\kappa T^2\beta^2 a'}{\pi(C\rho r_0^2\omega)^2}\frac{\eta+\eta^{-1}}{2\eta^2}\left[1+\frac{2k^2r_0^2\eta}{(\eta+\eta^{-1})(1+(2kl_{\mathrm{th}})^4)}\right], \tag{1}$$

where k_{B} is Boltzmann's constant, κ the thermal conductivity, T the temperature, $\beta = \partial n/\partial T$ where n is the refractive index, C the specific heat, ρ the density, r_0 is the beam radius defined in terms of the beam intensity, and ω is the angular frequency. The beam size is often defined as the beam radius for the amplitude distribution, $w_0 = \sqrt{2}r_0$. In this document, we employ the beam radius for the amplitude distribution, w_0, as the beam size. η, called ellipticity, is the ratio between the long and the short semiaxis of the elliptical beam within the BS, and it is given by [2, 3]

$$\eta = \frac{1}{\cos\psi}\sqrt{1-\left(\frac{\sin\psi}{n}\right)^2}, \tag{2}$$

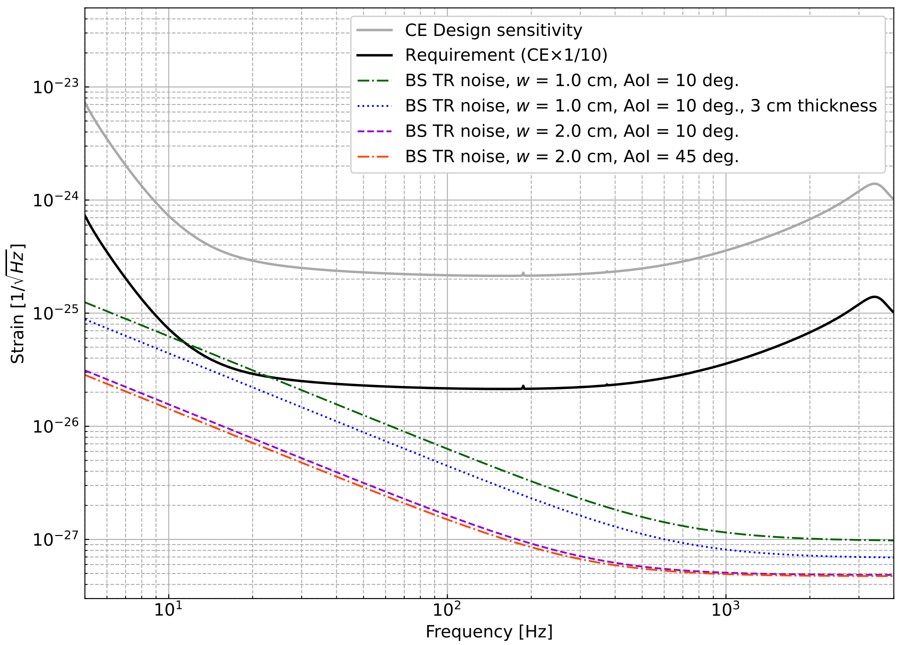

Figure 2: The requirement on the TR noise (black solid line), and the TR noise in the BS (green dash-dotted, blue dotted, dashed purple, and dash-dotted orange lines). Here the requirement is set to $1/10$ of the design sensitivity [5]. The blue line corresponds to the case of the BS thickness is 3 cm. Except for the blue line, we assume that the BS thickness is the same as that of aLIGO (i.e. 6 cm) [6].

where ψ the angle of incidence (AoI). $l_{\rm th} = \sqrt{\kappa/(C\rho\omega)}$ is the thermal diffusion length. a' is the optical path length in the BS which is given by [3]

$$a' = \frac{na}{\sqrt{1-\left(\frac{\sin\psi}{n}\right)^2}}, \tag{3}$$

where a is the thickness of the BS. Detailed derivation of Eq. (1) can be found in Refs [4, 1, 2]. It should be noted that Eq. (1) is in the units of signal recycling displacement. The TR noise in the strain sensitivity can be obtained by

$$\sqrt{S_h^{\rm TR}(\omega)} = \frac{\pi}{2\mathscr{F}}\frac{\sqrt{S^{\rm TR}(\omega)}}{L}. \tag{4}$$

Here, $\mathscr{F}$ is the arm cavity finesse, and L is the arm cavity length.

3 Result

Fig. 2 shows the design sensitivity of Cosmic Explorer and the TR noise in the BS under several different conditions. Considering from the estimated TR noise curves, the dependence on the angle of incidence is weak. Therefore, the TR noise does not change drastically even with the narrow angle of incidence [1]. For these cases, above around 500 Hz, the noise becomes flat. This is because the standing wave introduces a modulation of the inserted heat along the optical path with a characteristic period of $2k = 4\pi n/\lambda$ and it is of the same order as the thermal diffusion length, l_{th} [2]. Below this frequency, the noise shows a decrease of $1/\omega$.

When the beam size becomes about 1.0 cm and the BS thickness is 6 cm, the TR noise can become one of the limiting noise sources around 15 Hz. As long as the beam size is larger than ~ 2.0 cm, it is well below the design sensitivity even with smaller beam size compared to that of aLIGO (5.3 cm) [6]. If we reduce the beam size to 1.0 cm, the BS thickness needs to be below about 3.0 cm to keep the TR noise well below the design sensitivity.

At this frequency range, i.e., below ~ 100 Hz, the effect of standing wave can be negligible. Therefore, Eq. (1) can be approximated as

$$S^{\mathrm{TR}}(\omega) \approx \frac{4k_{\mathrm{B}}\kappa T^2\beta^2 a'}{\pi(C\rho r_0^2\omega)^2}. \tag{5}$$

In addition, for the case of narrow angle of incidence, a' can be approximated as

$$a' \approx na. \tag{6}$$

From above, the TR noise in the strain sensitivity below ~ 100 Hz becomes

$$\sqrt{S_h^{\mathrm{TR}}(\omega)} \approx \frac{\pi}{2\mathscr{F}L}\sqrt{\frac{4k_{\mathrm{B}}\kappa T^2\beta^2 na}{\pi(C\rho r_0^2\omega)^2}}. \tag{7}$$

By substituting parameters, one can get

$$\sqrt{S_h^{\mathrm{TR}}(\omega)} \approx 3.0\times10^{-26}\times\left(\frac{450}{\mathscr{F}}\right)\times\left(\frac{1.0\ \mathrm{cm}}{w_0}\right)^2\times\left(\frac{a}{3.0\ \mathrm{cm}}\right)^{1/2}\times\left(\frac{15\ \mathrm{Hz}}{f}\right), \tag{8}$$

where $f(=\omega/2\pi)$ is the frequency. Here we normalized at a frequency of 15 Hz where the TR noise can become the limiting noise source of CE. At 15 Hz, the requirement is about 3.6×10^{-26} $[1/\sqrt{\mathrm{Hz}}]$. Assuming that the beam size is $w_0 = 1.0$ cm, we can adjust the arm cavity finesse and the BS thickness such that

$$\left(\frac{450}{\mathscr{F}}\right)\times\left(\frac{a}{3.0\ \mathrm{cm}}\right)^{1/2} \lesssim 1.2. \tag{9}$$

Parameters which are used to compute the TR noise are summarized in Tab 1.

4 Backwards case

As shown in Fig. 3 and the reference [7], the BS could be used backwards to reduced the optical losses in the signal extracting cavity. In this case, there are two crossing standing waves inside the BS, and they can affect the TR noise level.

[1] Since the ellipticity η is of the order of unity, the standing wave effect term in Eq. (1) is dominated by $r_0^2/(1+(2kl_{\mathrm{th}})^4)$.

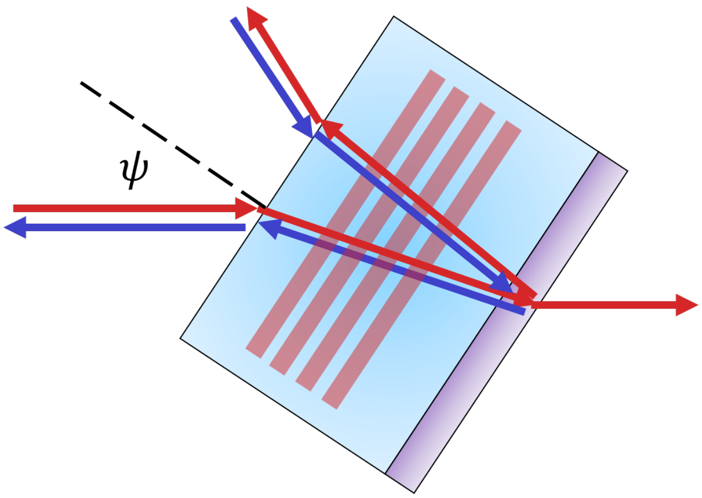

Figure 3: Schematic of a flipped beamsplitter.

5 Summary and discussion

The thermorefractive noise in the beamsplitter of CE will be well below the design sensitivity even with smaller beam size and angle of incidence. The initial estimation indicates that the beam size can be reduced to about 2.0 cm assuming the same thickness beamsplitter as aLIGO is used. The TR noise can be reduced by using a thinner BS, and that allows us to use 1.0 cm beam size on the BS.

The BS can be flipped to reduce the optical losses in the signal extracting cavity. The TR noise with this configuration will be estimated in future.

In addition to the TR noise, the substrate Brownian noise of the BS need to be evaluated as shown in the reference [8]. In order to compute the substrate Brownian noise, FEM analysis would be needed [8].

A Parameters

Tab. 1 show the list of parameters used to estimate the TR noise.

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Table 1: Parameter values [1].

Symbol	Description	Value
k_{B}	Boltzmann's constant	1.38×10^{-23} J/K
κ	Thermal conductivity	1.38 W/(m · K)
T	Temperature	300 K
n	Refractive index	1.45
β	Thermo-optical parameter	8.5×10^{-6} 1/K
a	Thickness of the BS [6]	6.0×10^{-2} m
k	Wave vector	8.56×10^{6} 1/m
C	Specific heat	746 J/(kg · K)
ρ	Density	2200 kg/m^3
L	Arm cavity length	40 km
$\mathscr{F}$	Arm cavity finesse	450

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