Component Interferometer Length Scaling for Network Science Metrics

https://dcc.cosmicexplorer.org/T2300001

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Contents

1 Introduction

In this document, we explore the scaling of reference science metrics with the geometry of a new facility, when a detector is added to an existing network of comparable sensitivity. The goal is to determine the appropriate power α of a scaling relationship of the form [science] $\propto L^{\alpha}$ for armlength L.

This scaling relationship is desired to allow comparison between different interferometer geometry proposals: for example, if a proposed XG site geometry has 1.5× another's armlength, how much "better" will the resulting XG network be? A simple scaling metric can be used to quickly evaluate cost tradeoffs for variant scenarios.

The relevant characteristic of the facility is for a two-arm detector the quantity $L \sin\theta$, with L the armlength and θ the opening angle. The primary impact of this characteristic is to change in the observatory's recorded signal amplitude for a given source.

$$A(f) \propto \frac{D_0}{D_L} \frac{L \sin\theta}{4\text{km}} A_0(f) \tag{1}$$

For a fixed source, this means a larger signal to noise ratio (SNR): $\rho \propto A$. Science metrics we consider here:

- SNR of fixed source - inversely proportional to measurement error for that source.
- Number of sources with a given loud SNR (sensitive volume in the nearby universe assuming uniform distribution).
- Redshift of most distant observable binary black hole: $z(D_H)$ with BBH horizon distance D_H. Roughly scales like $D_H \propto \rho_{\text{fixed}}$
- Total sensitive time-volume of BBH within that horizon.
- Redshift of most distant observable binary neutron star: $z(D_H)$ with BNS horizon distance D_H. Roughly scales like $D_H \propto \rho_{\text{fixed}}$
- Total sensitive time-volume of BNS within that horizon

For each of the metrics listed above, the relevant SNR is the standard integrated ratio of the expected signal amplitude squared to the power spectral density of noise in the interferometer over all frequencies, but a similar quantity can be constructed for specific measurement goals.

For each of these categories, we will look at the change in the metric described and fit that difference to a power-law scaling with the length of the new facility. For example, for the SNR of a fixed source, we write $\Delta\rho = \rho - \rho_0 \propto L_{new}^{\alpha}$ and fit for the coefficient α for the specific initial network ρ_0 and the total ρ after adding a new interferometer with length L_{new}.

Detectors will also have a source-specific factor F which depends on how well the frequencies containing signal information are measured by the instrumentation installed in the observatory, such that

$$\frac{\rho_{\mathrm{target}}}{\rho_{0,\mathrm{target}}} = F\frac{D_0}{D}\frac{L\sin\theta}{4\mathrm{km}} \tag{2}$$

2 Adding a detector to a network

Premise: Each component observatory will contribute to the total SNR of the signal in the detector network in quadrature. Therefore, impact from a new detector with signal SNR ρ is:

$$\Delta\rho = \sqrt{\rho_0^2 + \rho^2} - \rho_0 \tag{3}$$

If $\rho_0 \ll \rho_{\mathrm{new}}$, network contribution $\simeq \rho_{\mathrm{new}} - \rho_0$. If $\rho_0 > \rho_{\mathrm{new}}$, network contribution $\simeq \frac{1}{2}\rho_{\mathrm{new}}^2/\rho_0$. The transition between regimes is shown in Figure 1.

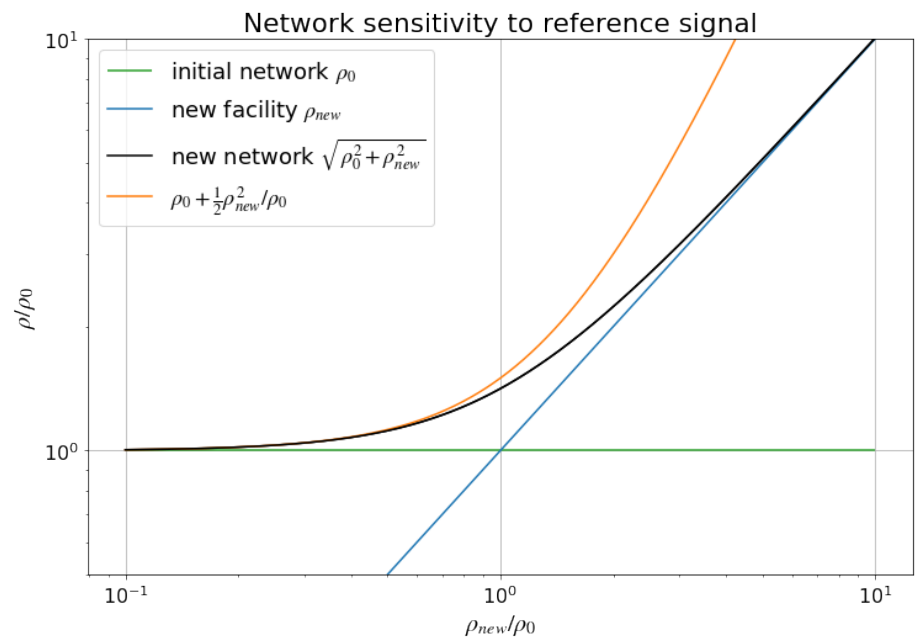


Figure 1: The impact of a new facility which measures a source with SNR ρ_{new}, added to an existing network which measures a source with SNR ρ_0, on the total network sensitivity $\rho = \sqrt{\rho_0^2 + \rho_{\mathrm{new}}^2}$.

3 Assumptions for the facility under consideration

We focus the scaling estimates on candidate configurations with $L_{\rm new} \sin\theta_{\rm new}$ that range from $(3-6)L_{LIGO}$ are being considered. They are being added to a network of two other detectors of comparable strength; this means we expect $\rho_0 = \sqrt{2}\rho_{\rm new} \simeq (5.7-8.5)\rho_{LIGO}$.

We measure results here in "LIGOs." However, an XG facility of the same length will tend to have larger SNR due to wider sensitive band. We scale the resulting SNR of the XG observatory network by a phenomenological amplitude factor F. This F factor is source-specific, and is approximated here as a fixed value based on existing horizon changes. If $F = 1$, then $\rho/\rho_{LIGO} = L/L_{LIGO}$.

It is straightforward to re-check these constants for other ranges of $\rho_{\rm new}$ and ρ_0, or to modify the reference values for the estimate of F.

4 Metrics for nearby signals

These results "in LIGO" scale directly with F, so we take $F = 1$.

$$\frac{\rho_{\rm target}}{\rho_{0,\rm target}} = F\frac{D_0}{D}\frac{L\sin\theta}{4\rm km} \tag{4}$$

4.1 Strength of reference signal

Science metric: Change in signal-to-noise for a fixed signal.

New network sensitivity $\Delta\rho = \sqrt{\rho_0^2 + \rho_{new}^2} - \rho_0$ in LIGOs. These would scale also with F.

This scaling is shown in Figure 2. It should apply to most sources (not only transients).

4.2 Number of nearby signals above given SNR

Science metric: Number of loud signals

The metric for some population-level science goals will be proportional to number of signals observed with network SNR above a fiducial value, where signals are in nearby universe (see cosmology below for limitations of this scaling). So we expect $N_0 \propto V_0 \propto A_0^3 \propto \rho_0^3$. Write resulting change $V - V_0$ in units of V_{LIGO}. These would scale also with F^3.

This metric would apply to sources with uniform volumetric distribution in the relevant distance range.

This scaling is shown in Figure 3.

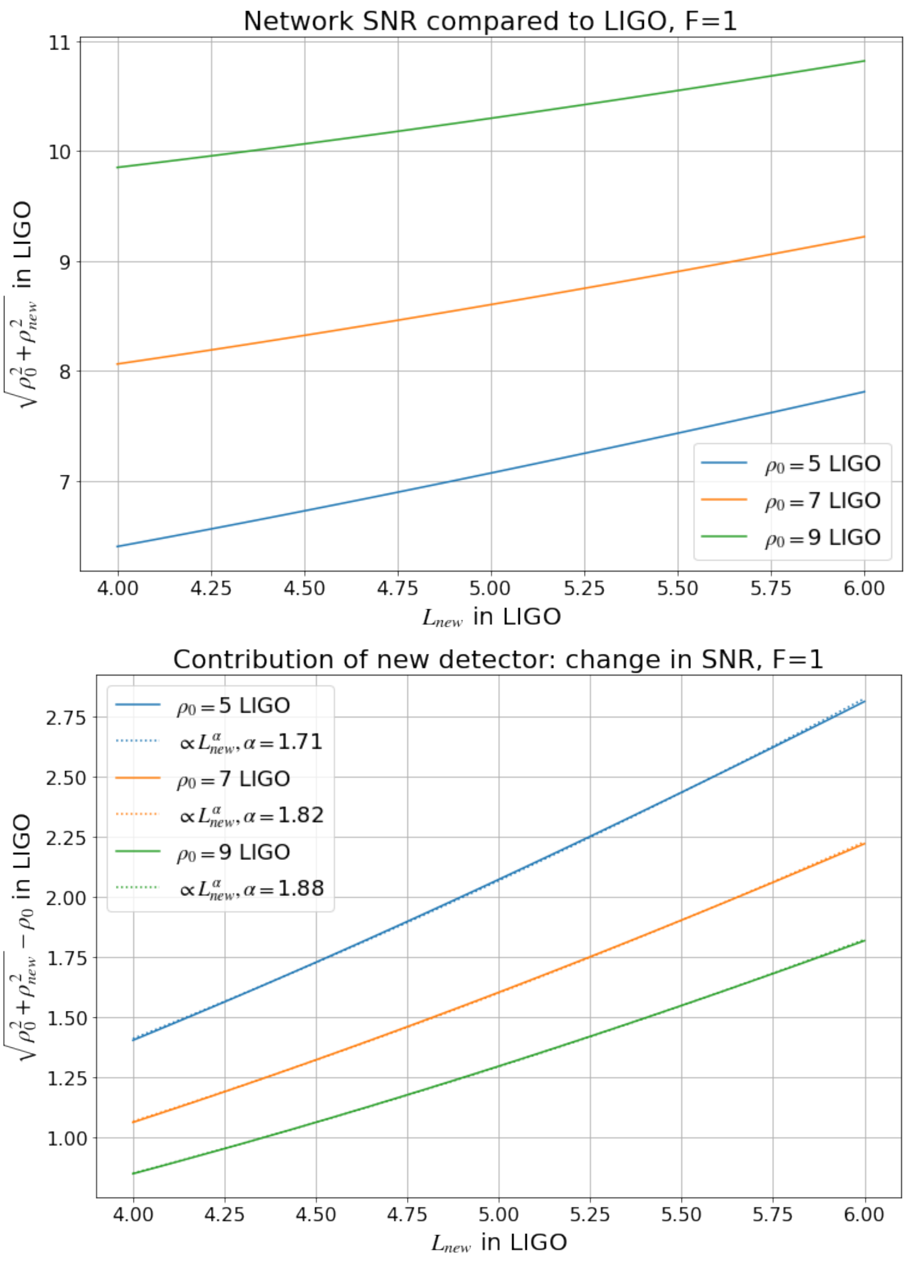
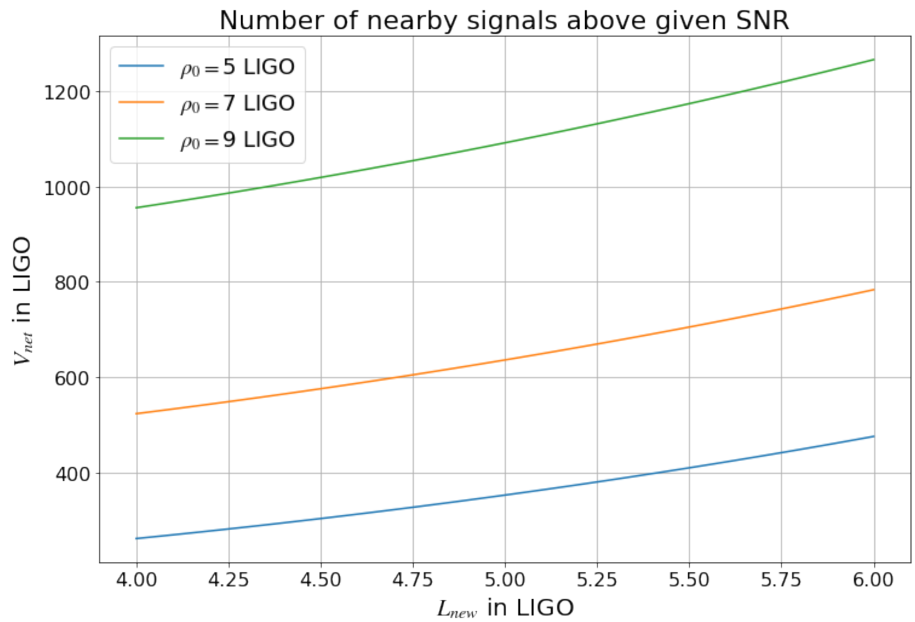


Figure 2: The impact of a new facility with arm length $L_{\rm new}$ that scales up LIGO geometry, added to an existing network which measures a source with SNR ρ_0, on the total network sensitivity $\rho = \sqrt{\rho_0^2 + \rho_{\rm new}^2}$.



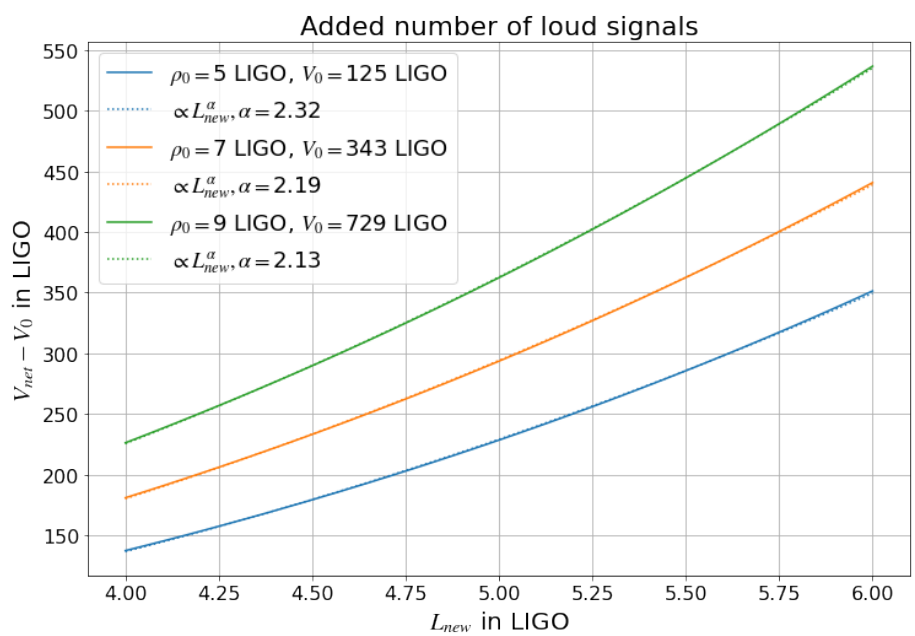


Figure 3: The impact of a new facility with arm length $L_{\rm new}$ that scales up LIGO geometry, added to an existing network which measures a source with SNR ρ_0, on the sensitive volume. F is a factor which could capture instrumental differences beyond length scaling for the new facility.

5 Sources at cosmological distances

The total volume of universe that the observatory can observe can be found by integrating differential comoving volume dV_c/dz versus redshift z out to thehorizon redshift z_H. The rate of events must also be scaled from a local comoving rate $\dot{n}$ to the rate in the observer frame. This gives total observed event rates $\propto$ local rate $\times$ differential volume $\times$ inverse scale factor $\times$ detection probability[`https://arxiv.org/abs/1904.10976`]

$$R_D(z) = \frac{dV_c}{dZ}\frac{\dot{n}}{1+z}P_{det}(z) \tag{5}$$

We assume a instant-off threshold for $P_{det}(z)$ at the the horizon redshift $z(D_H)$, and define VT to scale like the observe rate if there was a constant comoving rate $\dot{n}$. The VT as a function of D_H is therefore given by integrating differential comoving volume divided by $1+z$ out to redshift $z(D_H)$. For nearby sources, this is as above $\propto D_H^3$. However, for cosmological distances this scaling breaks down, as shown in Figure 4.

Since the scaling depends on the horizon, we need to map between the length scale of the proposed facility under consideration. This requires a more concrete estimate for the F above.

For the reference sources here, we estimate the frequency factor F from prior results for the horizon distance of different observatories, We use Cosmic Explorer (CE) in the horizon study as the representative instrumental configuration comparable to LIGO, a detector length of $10L_{LIGO}$ should give the horizon improvement from A+ to CE in horizon study, from z_{APLUS} to z_{CE}

If we estimate the improvement in horizon purely from length scaling we expect $10L_{LIGO}$ to give a horizon luminosity distance of $10D_{H,LIGO}$. We set F so that $z(F \times 10D_{H,LIGO}) = z_{CE}$.

For sources at cosmological distance, the F will change slightly across the range of candidate $L_{\rm new}$ due to varying the redshift of the horizon source signal. We neglect this effect in this version of the scaling estimate and hold F constant across proposed XG facilities.

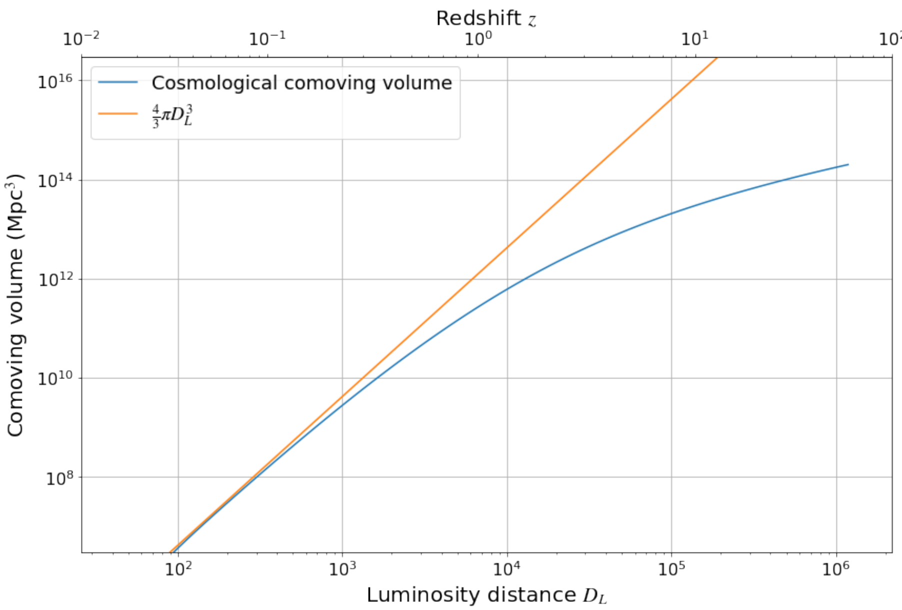


Figure 4: The breakdown of VT scaling like D_H^3. This combines two cosmological factors — the impact of the size of the early universe on the change in comoving volume as horizon redshift increases, and the impact of the local scale factor on the ratio of the rate of signals per comoving cosmological volume to the rate of observed signals on Earth.

6 Metrics for source populations

6.1 BBH motivated

Metrics: Horizon for BBH, cosmological VT for BBH.

Relevant amplitude-only scaling for the horizon (redshift of furthest signal) for BBH, and the integrated volume-time VT for all BBH.

Specifically we pick a 30-30 $M_{\odot}$ BBH as reference and estimate the horizon change from APLUS at $z_H = 1.5$ to CE at $z_H = 60$. This gives $F \simeq 4$.

Since $z \sim D_L$, the horizon distance scales similarly to the SNR of a reference signal. This is demonstrated in Figure 5; the change in horizon scales much like the fiducial signal SNR.

Since the accumulation of VT is slower with z in the early universe, the sensitive volume for BBH scales at a lower power with L_{new} compared to VT in the local universe. This scaling is shown in Figure 6.

6.2 BNS motivated

Metrics: Horizon for BNS, cosmological VT for BNS.

Relevant amplitude-only scaling for the horizon (redshift of furthest signal) for BBH, and the integrated volume-time VT for all BBH.

Specifically we pick a 1.4-1.4$M_{\odot}$ BBH as reference and estimate the horizon change from APLUS at $z_H = 0.1$ to CE at $z_H = 3$. This gives $F \simeq 4$.

Since $z \sim D_L$, the horizon distance scales similarly to the SNR of a reference signal. This is demonstrated in Figure 7; the change in horizon scales much like the fiducial signal SNR.

Since the accumulation of VT at cosmological distances is moderately slower than the local universe, so the sensitive volume for BNS scales at a moderately lower power with L_{new} compared to VT in the local universe. This scaling is shown in Figure 2.

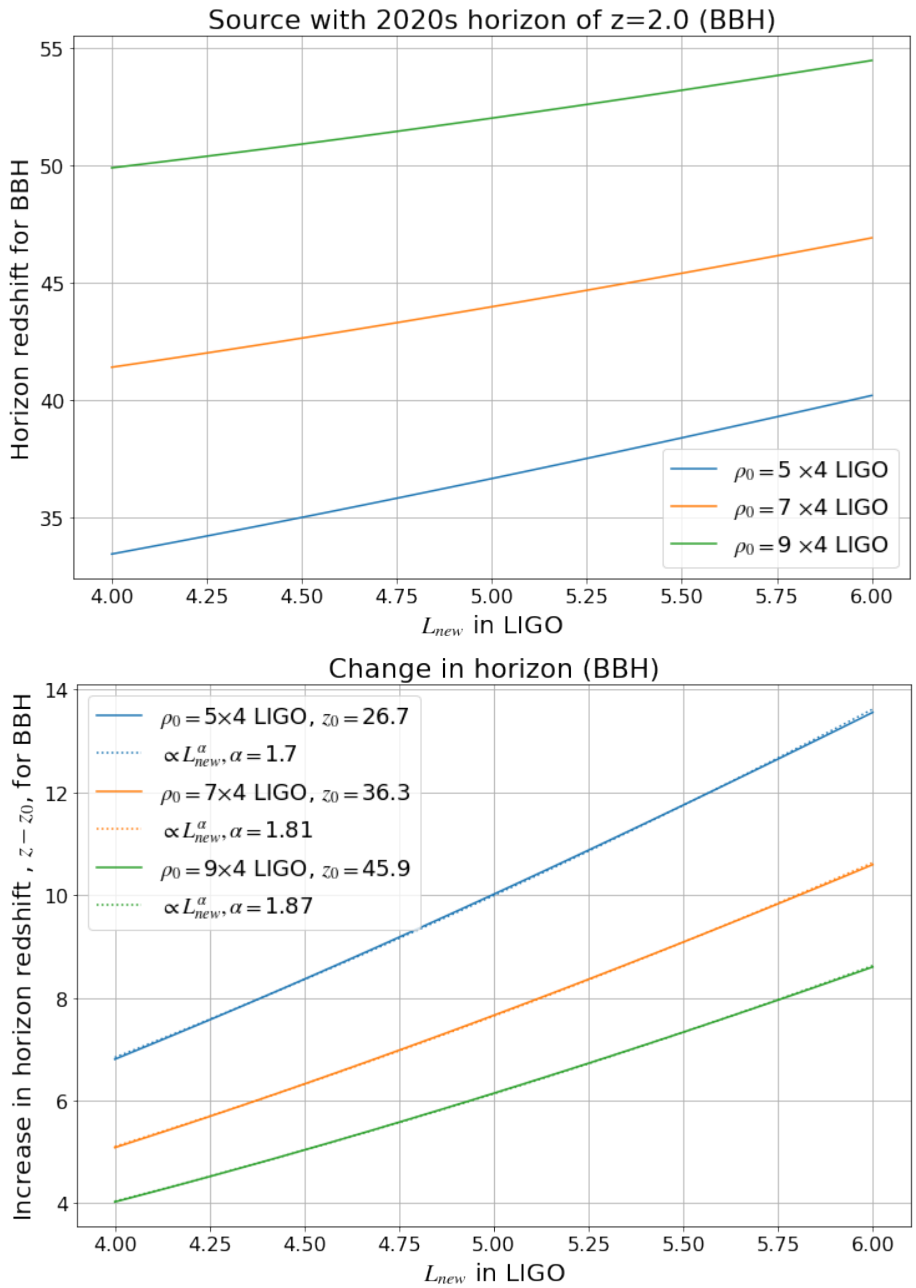
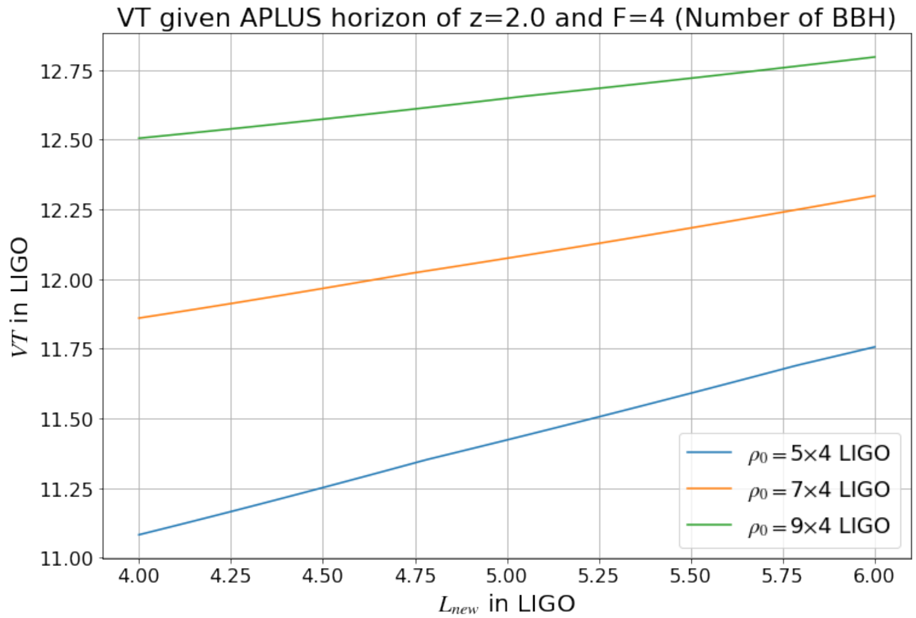


Figure 5: The impact of a new facility with arm length $L_{\rm new}$, added to an existing network which measures a source with SNR ρ_0, on the horizon redshift for BBH.



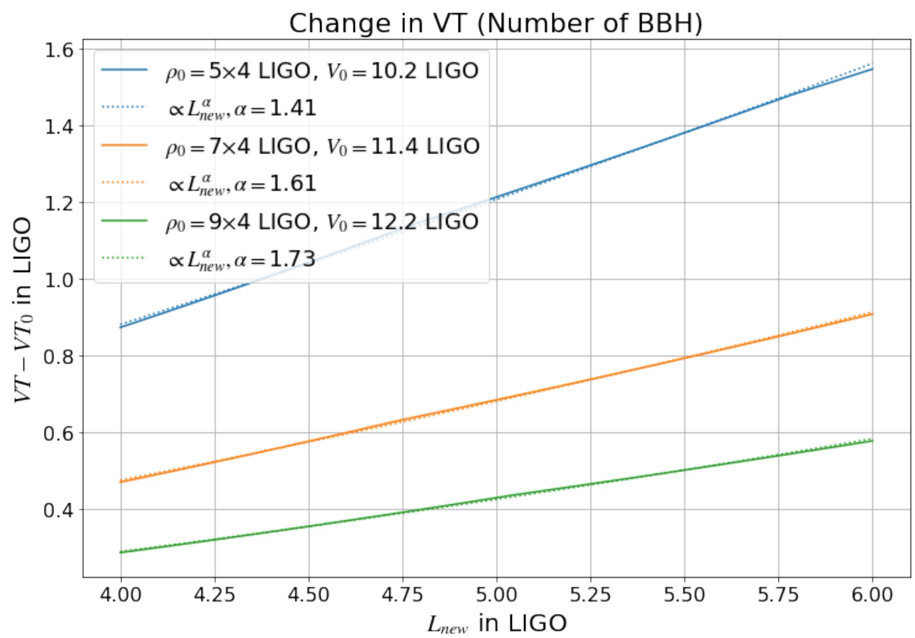


Figure 6: The impact of a new facility with arm length L_{new}, added to an existing network which measures a source with SNR ρ_0, on the sensitive VT for BBH.

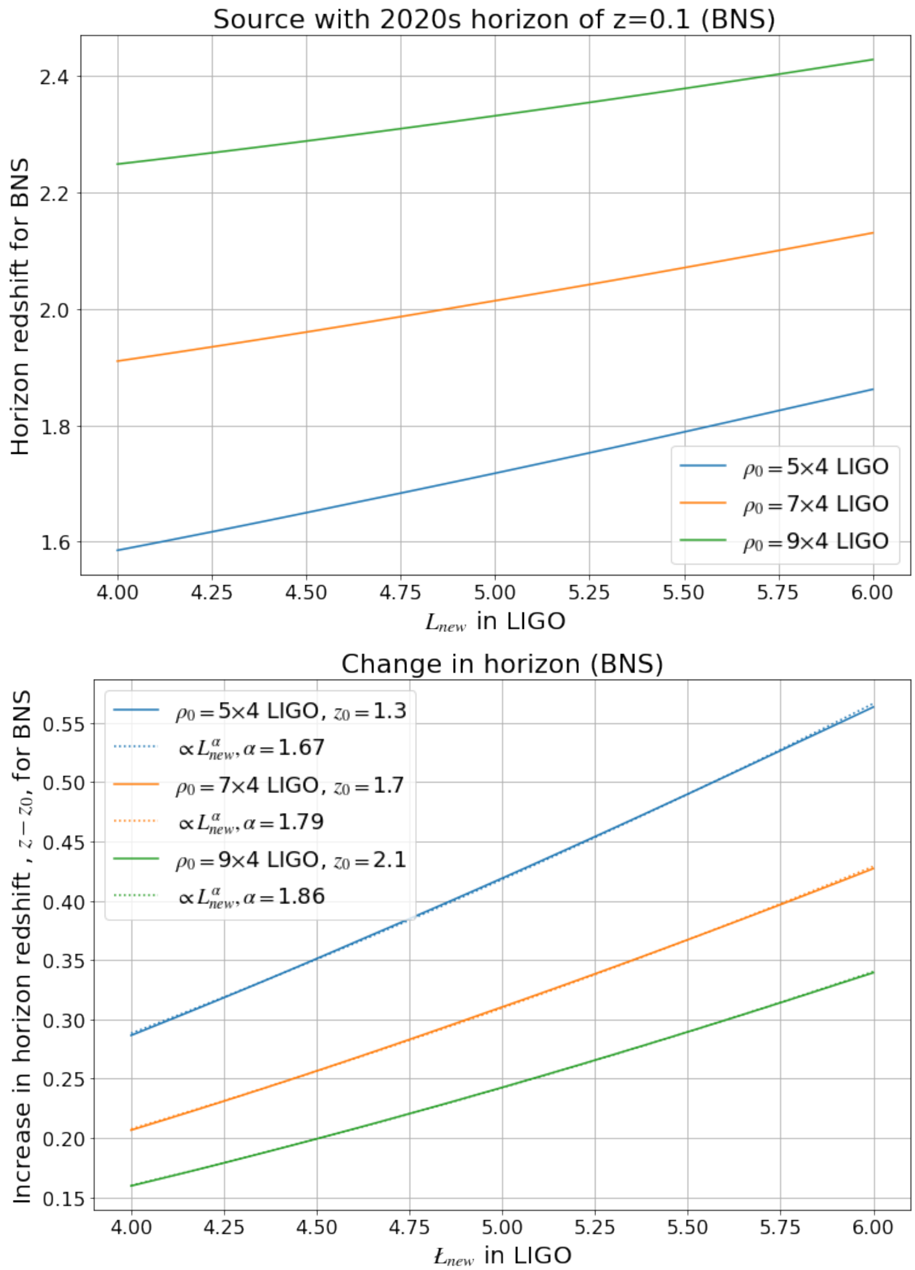


Figure 7: The impact of a new facility with arm length L_{new}, added to an existing network which measures a source with SNR ρ_0, on the horizon redshift for BNS.

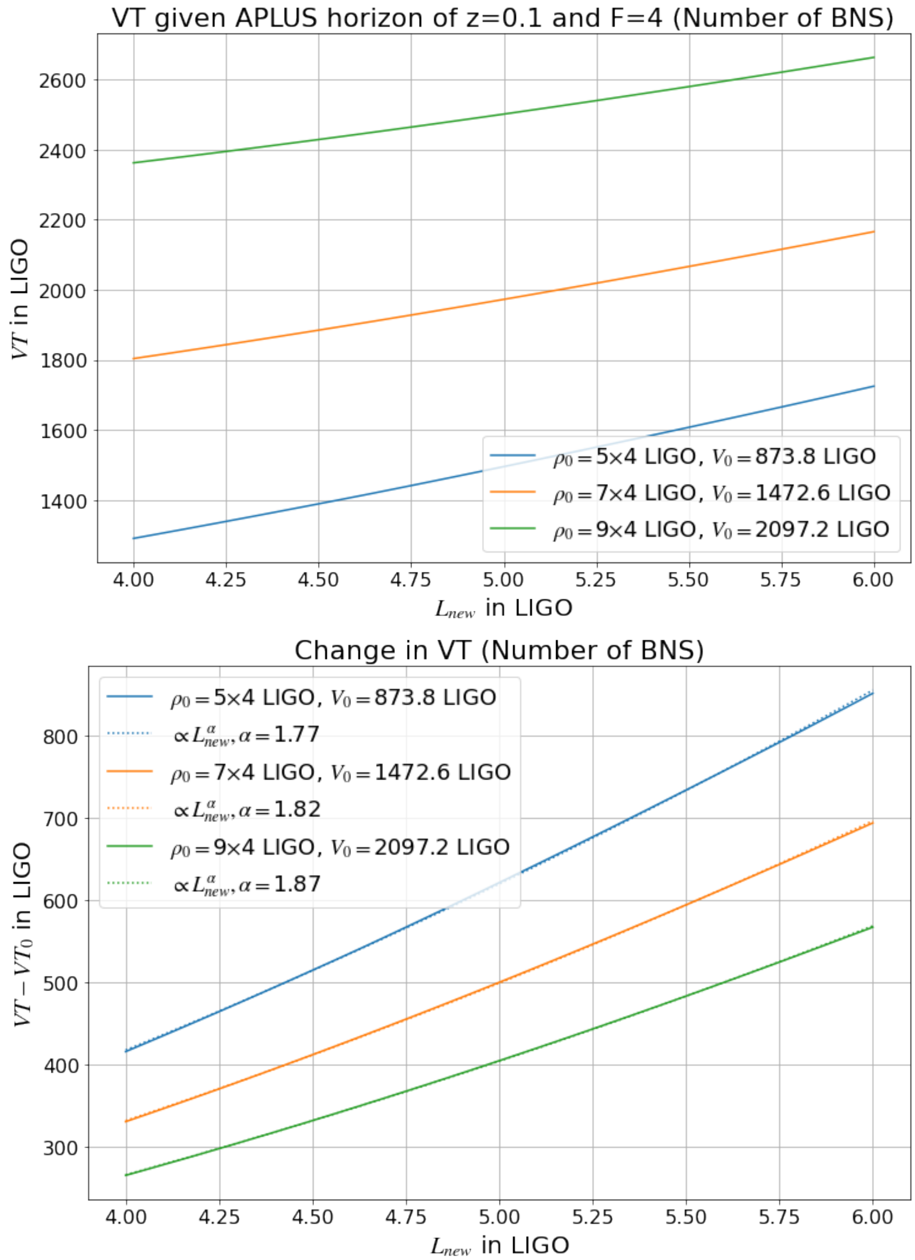


Figure 8: The impact of a new facility with arm length L_{new}, added to an existing network which measures a source with SNR ρ_0, on the sensitive VT for BNS.